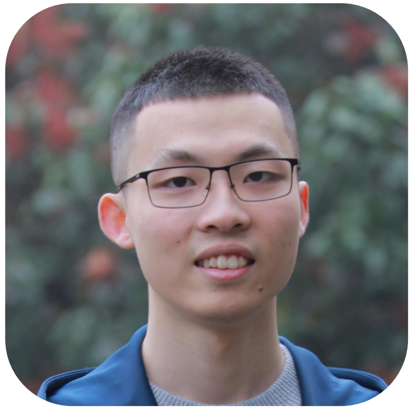
Bidder Selection Problem in Position Auctions: A Fast and Simple Algorithm via Poisson Approximation





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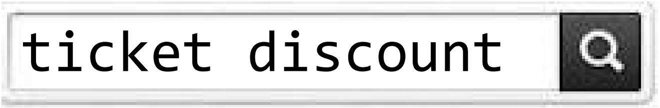
Bidder Selection in Online Ad Auction

- Online ad auction
  - Ad company sells ad slots to advertisers
  - Real time and automated

- Bidder selection
  - Bidders' valuations are computed from a ML model
  - Running the model for all bidders: costly and slow
  - A prior distribution for each bidder is available
  - **Two-stage selection:** filter out a fraction of bidders, then run auction

Example Scenario



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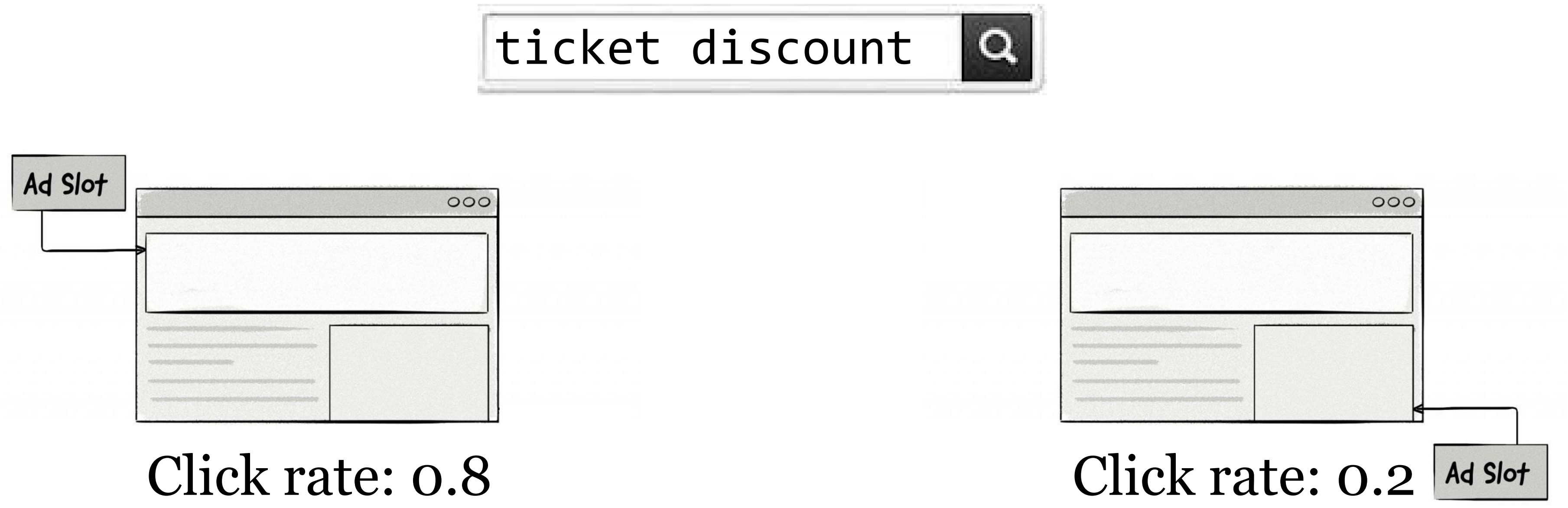

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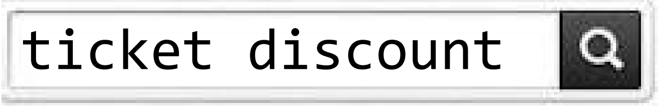

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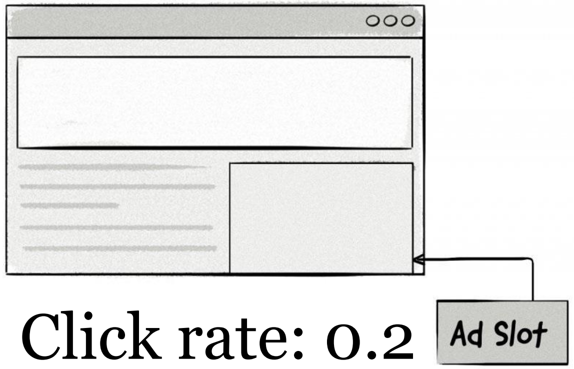

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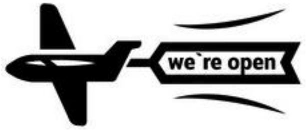


Value: 5$

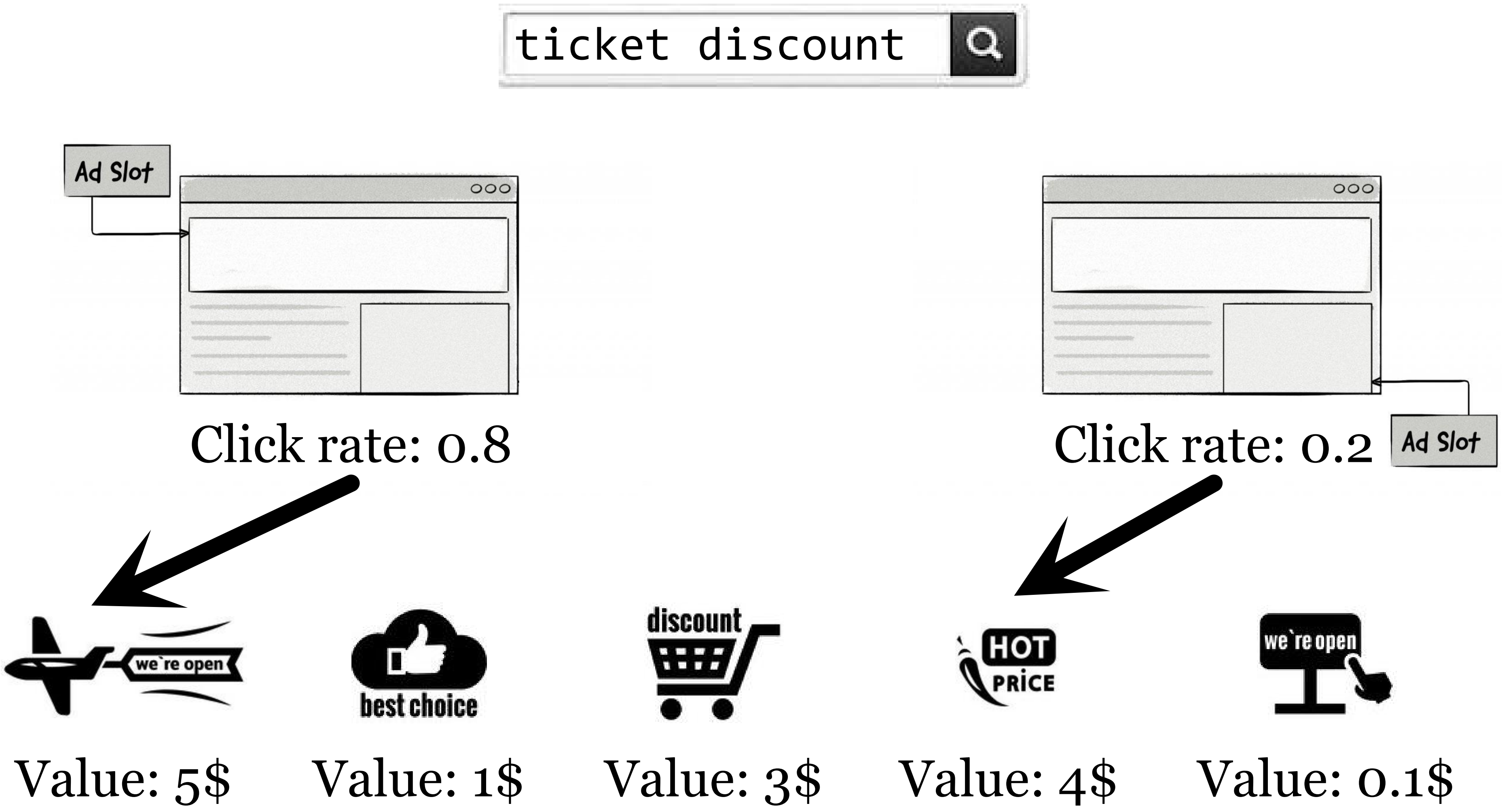
Value: 1$

Value: 3$

Value: 4$

Value: 0.1$

Example Scenario



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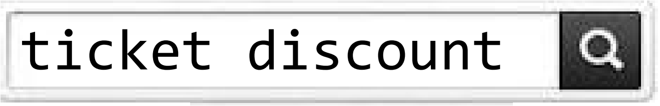

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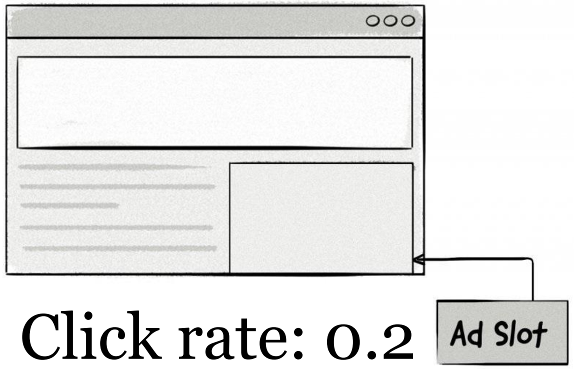

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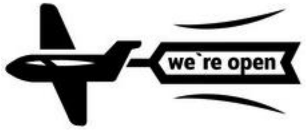


$v_1 \sim D_1$

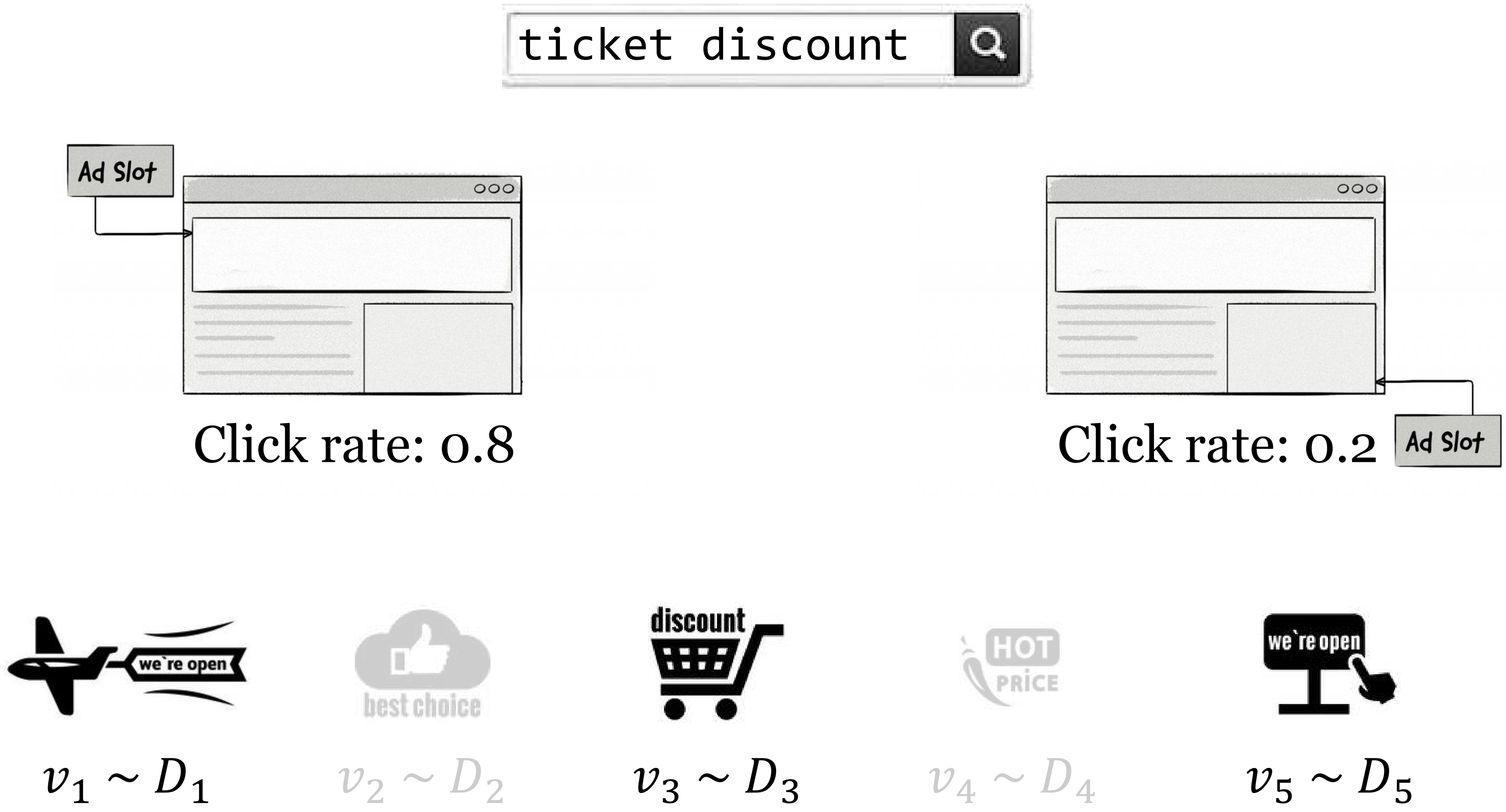
$v_2 \sim D_2$

$v_3 \sim D_3$

$v_4 \sim D_4$

$v_5 \sim D_5$

Example Scenario



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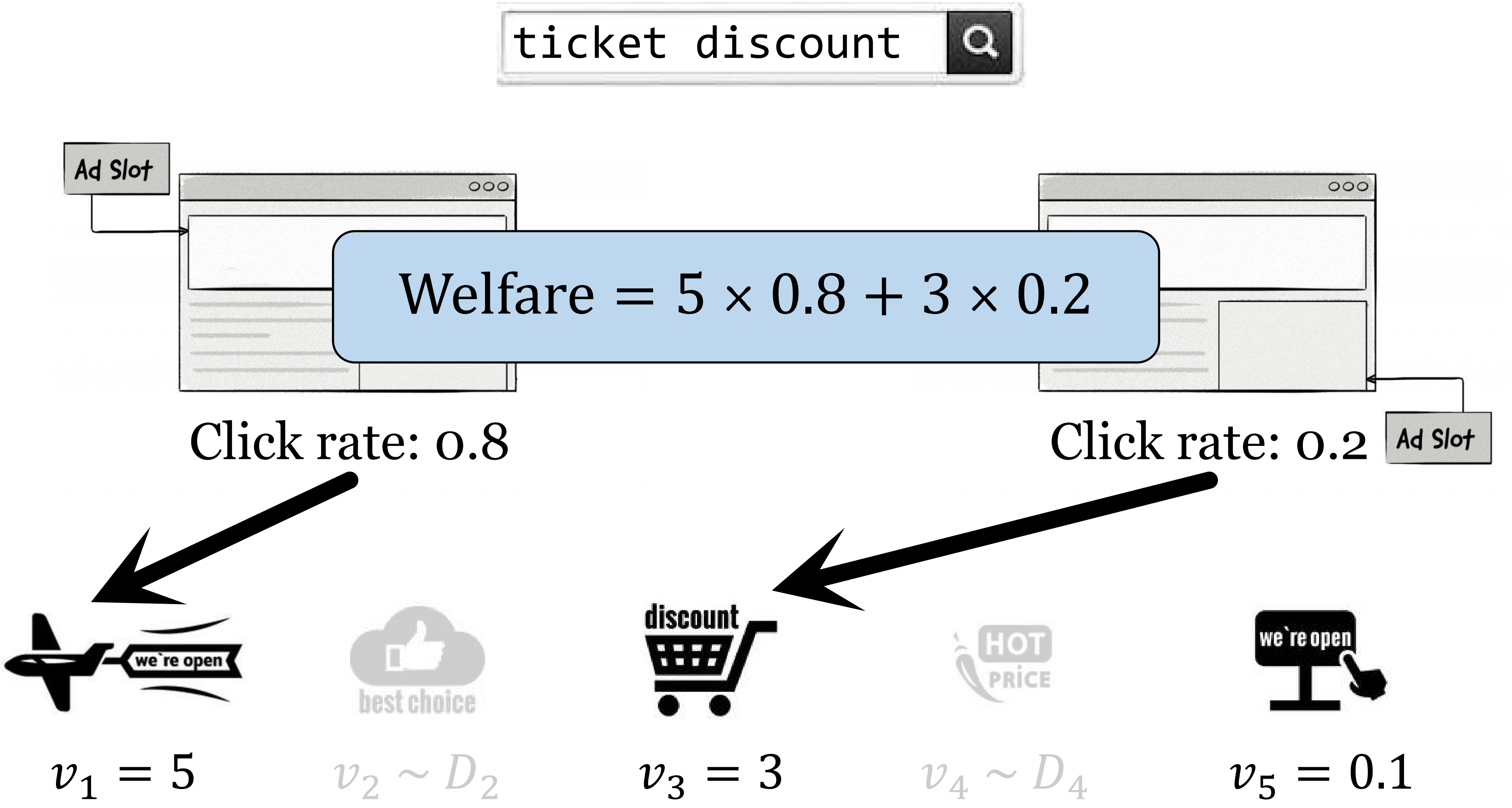


Bidder Selection Problem (Single-Item)

- There are ***n*** **bidders** competing for an ad slot
  - Bidder i has value $v_i \sim D_i$ from an independent, known distribution

- We need to **choose *k* bidders**, maximizing

$$\mathbb{E}_{v_1,\ldots,v_n}[\max\{v_i \mid \text{bidder } i \text{ is chosen}\}]$$

- Exact optimum is NP-hard; aim for **$(1-\varepsilon)$-approximation**

Bidder Selection Problem (Position Auction)

- There are n **bidders** competing for **some ad slots**
  - Bidder i has value $v_i \sim D_i$ from an independent, known distribution
  - **There is a non-negative weight sequence $w_1 \geq w_2 \geq \cdots \geq w_k$**
- We need to **choose k bidders**, maximizing

$$\mathbb{E}_{v_1,\ldots,v_n}\left[\sum_{i=1}^{k} v_{(i)} w_i\right]$$

where $v_{(i)}$ is the i-th largest value among k chosen bidders

Previous Results on BSP

- Previous $(1-\varepsilon)$-approximation (PTAS) algorithms on BSP:
  - [CHLLLL2016], [MNPR2020]: For **single-item auction**
  - [SS2021]: For ***L*-unit auctions** (i.e., position auctions with $w_i \in \{0, 1\}$)

- All of them are based on discretizing all possible distributions
  - Bad dependency on ε

$$2^{O(1/\varepsilon)^{O(1/\varepsilon)}}$$

  - Take years for small instances like $n = 3, k = 2, \varepsilon = 0.2$
  - Not implementable in practice

Our Results

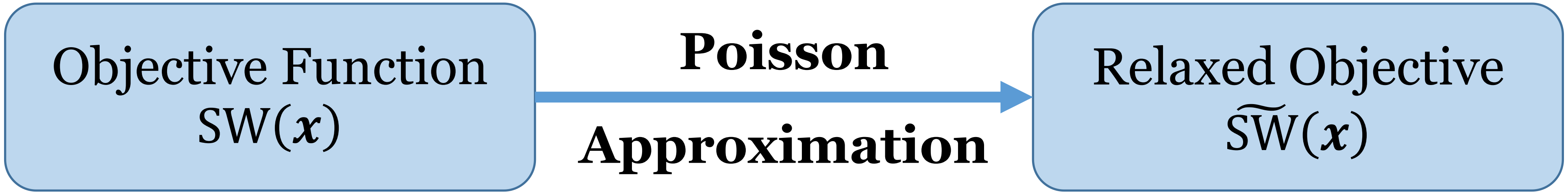
- There is a **polynomial-time** algorithm for BSP choosing k bidders out of n with approximation ratio

$$1 - O\left(k^{-1/4}\right)$$

  - This implies a PTAS for BSP for general position auctions

- The algorithm is **easily implemented**, runs **fast** and obtains **high-quality solutions** in experiments

Main Technique: Poisson Approximation



- Relaxed objective $\widetilde{\mathrm{SW}}(\boldsymbol{x})$ has 3 merits:
  1. **Good approximation** ratio: $1 - O\left(k^{-1/4}\right)$
  2. **Convex**, thus easy to optimize
  3. **Works** for general **position auctions** (not only single-item)

Algorithm Framework

1. Poisson approximation gives the **relaxed objective** $\widetilde{\mathrm{SW}}(\boldsymbol{x})$

2. Run **convex optimization** to find (a fractional solution) $\boldsymbol{x}$ that maximizes $\widetilde{\mathrm{SW}}(\boldsymbol{x})$

3. Use **rounding** techniques to transform $\boldsymbol{x}$ to an integer solution

Experiments

- We test homebrew implementations of 3 algorithms (using python + standard convex libraries)
- On large instances ($n = 1000$, $k = 200$):

	Local Search	Greedy	Our Algorithm
Running Time	$>$ 1 week	1 day	45 sec
(Relative) Welfare	N/A	97.38%	100.00%

- On all test cases, our algorithms shows **$>$ 99% approximation** compared to the benchmarks (Local Search & Greedy)

Future Directions

- Bidder Selection Problem under different feasibility constraints
  - E.g., matroid, matching, and intersection of matroids
- Revenue maximization for other auction formats
- Improve the approximation ratio $1 - O\left(k^{-1/4}\right)$