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# Not All Rollouts are Useful: Down-Sampling Rollouts in LLM Reinforcement Learning

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## Abstract

Reinforcement learning with verifiable rewards (RLVR) has emerged as a powerful paradigm for enhancing reasoning capabilities in large language models. However, it is constrained by a fundamental asymmetry in computation and memory requirements: rollout generation is embarrassingly parallel and memory-light, whereas policy updates are communication-heavy and memory-intensive. To address this, we introduce **PODS** (Policy Optimization with Down-Sampling). PODS produces numerous rollouts in parallel, then trains on only an informative subset, preserving learning signals while slashing update cost. We instantiate PODS with *max-variance down-sampling*, a principled criterion that maximises reward diversity and show it admits an  $O(n \log n)$  solution. Empirically, coupling PODS with Group Relative Policy Optimization (GRPO) achieves superior performance over standard GRPO across different reasoning benchmarks and hardware environments.

## 1 Introduction

Reinforcement learning with verifiable rewards (RLVR) has recently emerged as a powerful way to enhance the reasoning capabilities of large language models (LLMs) and has driven state-of-the-art performance on mathematical, coding, and general problem-solving benchmarks [1–4]. RLVR algorithms, including Proximal Policy Optimization (PPO) [5] and Group Relative Policy Optimization (GRPO) [6], share a two-phase structure: an *inference phase*, in which rollouts are generated and scored, followed by a *policy-update phase*, in which model parameters are updated using those scores.

These two phases place different computational demands on the hardware. Inference is embarrassingly parallel and has modest memory requirements, enabling modern accelerators to produce thousands of rollouts concurrently. In contrast, policy updates are communication-heavy and memory-intensive, requiring the storage of full-precision optimizer states and the synchronization of parameters and gradients across devices. Consequently, directly applying RLVR algorithms throttles inference, generating only as many rollouts as can be updated, leaving compute resources underutilized during inference. Practitioners therefore resort to memory-saving techniques such as gradient accumulation, gradient checkpointing, model sharding, or CPU offloading. While these methods reduce inference underutilization, they further increase the communication overhead during policy updates.

Our work provides a better solution. We observe that *not all rollouts contribute equally to model improvement*. Beyond a certain point, additional samples not only provide sharply diminishing returns, but also introduce noise into the training that can degrade the converging performance. By training on a strategically chosen subset, we can recover the wall-clock time otherwise spent on superfluous policy updates while strengthening the learning signal. We formalize this idea in **PODS** (Policy Optimization with Down-Sampling). As illustrated in Fig. 1, PODS maximizes hardware utilization during inference by generating a large pool of rollouts ( $n$  per prompt) but updating the policy on only the  $m < n$  informative examples selected by a principled down-sampling rule.

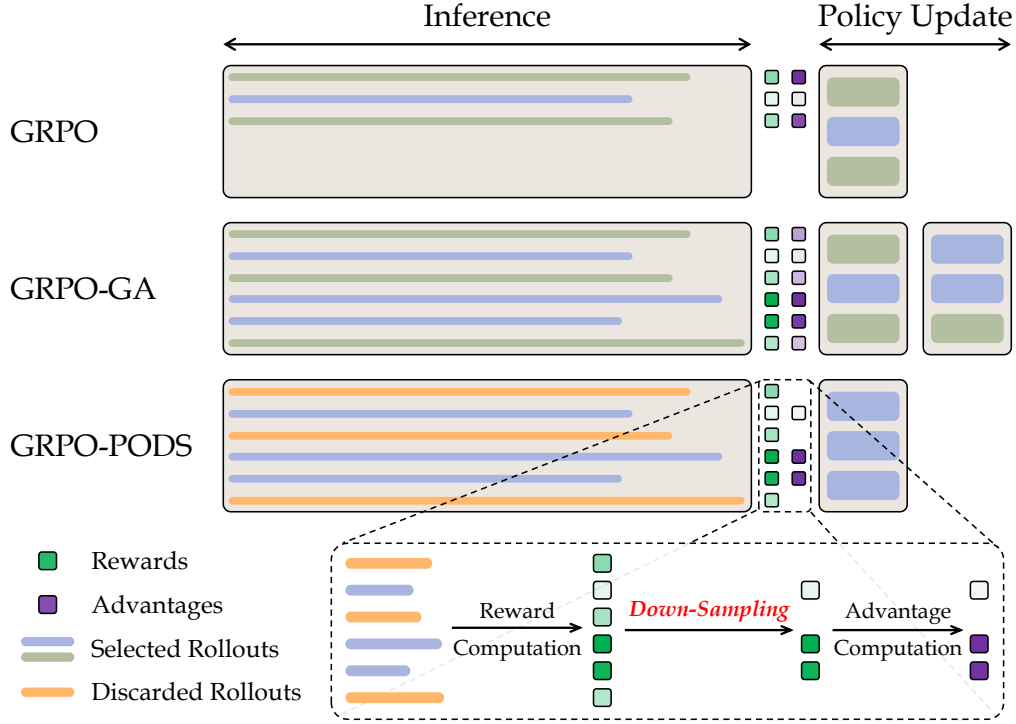


Figure 1: Visualization of three training strategies: standard GRPO, GRPO with gradient accumulation (GRPO-GA), and GRPO with PODS (GRPO-PODS). Standard GRPO generates  $n$  rollouts and trains on all of them, leaving inference hardware underutilized. GRPO-GA alleviates this issue with memory-saving techniques such as gradient accumulation, but at the cost of a more complex policy-update phase. In contrast, GRPO-PODS also generates  $n$  rollouts but trains on only  $m$  carefully selected examples, maximizing inference utilization, avoiding gradient-accumulation overhead, and providing a cleaner learning signal that yields better final performance.

Within the PODS framework, we introduce *max-variance down-sampling*, a theoretically motivated criterion that selects the subset of rollouts with the greatest reward variance, thereby preserving strong contrastive signals. We show that the resulting combinatorial problem can be solved in  $O(n \log n)$  time and, in the common binary-reward setting, reduces to picking the  $m/2$  highest-reward and  $m/2$  lowest-reward rollouts. Empirically, we evaluate GRPO with PODS on GSM8K [7] and MATH [8] across multiple hardware setups. GRPO with PODS achieves superior performance at convergence compared with standard GRPO, regardless of whether gradient accumulation is enabled.

## 2 Related Work

**Reinforcement learning for LLM reasoning.** Reinforcement learning has emerged as a powerful paradigm for enhancing the reasoning capabilities of LLMs across mathematical, coding, and problem-solving domains [1, 6, 9]. Although classical algorithms such as Proximal Policy Optimization (PPO) [5] laid the foundation, recent work has tailored them specifically to language models. Group Relative Policy Optimization (GRPO) [6] has gained prominence for reasoning tasks because of its implementation simplicity, competitive performance relative to PPO, and lack of a separate critic network. DeepSeek R1 [10], which leveraged large-scale RL, has sparked interest in reasoning-focused RL methods [11–14]. Meanwhile, value-based approaches like PPO remain central [15, 16], alongside with complementary techniques such as Monte Carlo Tree Search [17, 18] and multi-agent methods [19]. Two recent works share superficial similarities to ours: DAPO [20], which dynamically skips problems with binary accuracy during training, and VAPO [15], which advocates generating additional rollouts per problem instead of enlarging the batch size. However, neither method systematically *down-samples* the rollouts produced during the inference-to-update pipeline—our

key contribution. By tackling this computational-efficiency bottleneck, our approach complements existing methods and can be combined with them to further improve reasoning performance.

**Down-sampling and data selection.** The scale of modern machine learning necessitates effective data management strategies, particularly as datasets grow larger, noisier, and more imbalanced. Training on the full dataset can be prohibitively expensive, motivating sophisticated data-selection and down-sampling methods. Such techniques succeed across diverse settings—from theoretical results in clustering [21] and regression [22–24] to practical systems in speech recognition [25, 26] and computer vision [27, 28]. In reinforcement learning, prioritized experience replay [29] and related methods [30–32] highlight the value of selective sampling from experience buffers. More recently, careful data selection has become central to foundation-model training [33–35] and emerging applications such as computational advertising [36, 37]. Yet, to our knowledge, we are the first to apply principled down-sampling to the rollout-generation stage of LLM reinforcement learning, mitigating a key computational bottleneck while strengthening the learning signal.

### 3 Down-Sampling Rollouts in GRPO

In this section, we present our approach to resolving the computational asymmetry between inference and policy updates in LLM reinforcement learning. We first review the standard GRPO algorithm in Section 3.1, highlighting its *structural* components and computational demands. Next, in Section 3.2, we introduce the **PODS** (Policy Optimization with Down-Sampling) framework, which strategically selects informative rollouts to maximize hardware utilization during both inference and policy-update phases. In Section 3.3, we develop a theoretically grounded *max-variance down-sampling* method that preserves strong contrastive signals by retaining only rollouts from the extremes of the reward spectrum. We show that this method admits an elegant,  $O(n \log n)$  solution, making it practical for real-world deployment. Overall, our framework retains the advantages of GRPO while boosting computational and memory efficiency across diverse hardware setups.

#### 3.1 Preliminaries

Group Relative Policy Optimization (GRPO) [6] is a reinforcement-learning algorithm intended to enhance the reasoning capabilities of large language models (LLMs), particularly within the RLVR setting. Each GRPO training step follows a structured, two-phase process, described below.

**Inference phase.** Let  $\pi_\theta$  denote the policy parameterized by  $\theta$ , which defines a distribution over next-token probabilities given the previous tokens in a sequence. Given a single input prompt  $p$  (e.g., a math problem), GRPO first generates a group of  $n$  rollouts  $\mathbf{o} = (o_1, o_2, \dots, o_n)$  by autoregressively sampling from  $\pi_\theta$ . Each rollout is a complete token sequence excluding the prompt, representing a possible solution. Each rollout is then evaluated using a reward model  $r_i = R(o_i)$ , which scores the quality and correctness of the corresponding output  $o_i$ . This yields a reward vector  $\mathbf{r} = (r_1, r_2, \dots, r_n)$ . From these rewards, we compute normalized advantage estimates:  $a_i = (r_i - \mu)/\sigma$ , where  $\mu$  and  $\sigma$  are the mean and standard deviation of the rewards respectively.

**Policy update phase.** After computing the advantages, the policy is updated by optimizing the GRPO objective  $L_{\text{GRPO}}(\theta)$ . Specifically, for each rollout  $o_i$  with advantage  $a_i$ , we compute a loss for each token position  $t$ , and then average over all tokens and rollouts:

$$L_{\text{GRPO}}(\theta) = \frac{1}{n} \sum_{i=1}^n \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \min \left[ \frac{\pi_\theta(o_{i,t} \mid p, o_{i,<t})}{\pi_{\theta_{\text{fixed}}}(o_{i,t} \mid p, o_{i,<t})} \cdot a_i, \text{clip} \left( \frac{\pi_\theta(o_{i,t} \mid p, o_{i,<t})}{\pi_{\theta_{\text{fixed}}}(o_{i,t} \mid p, o_{i,<t})}, 1 - \epsilon, 1 + \epsilon \right) \cdot a_i \right].$$

where  $|o_i|$  is the number of tokens in  $o_i$  and  $\pi_{\theta_{\text{fixed}}}$  is a frozen copy of the policy used for importance weighting. This asymmetric loss embodies the *slow to adopt, quick to abandon* learning principle—limiting how aggressively the policy increases probabilities for tokens in high-reward rollouts while allowing more substantial reductions for low-reward sequences.

### 3.2 PODS Framework

We propose to *decouple the inference and training phases* in GRPO. Rather than updating on every generated rollout, PODS first produces  $n$  rollouts in parallel and then trains on only a smaller subset of size  $m < n$  selected by a down-sampling rule  $D$ . This strategy exploits parallelism during inference while substantially reducing the communication and memory costs of the subsequent policy update.

**Definition 3.1** (Down-sampling rule).  $D(\mathbf{o}, \mathbf{r}; m)$  is a function that takes as inputs  $n$  rollouts  $\mathbf{o} = (o_1, o_2, \dots, o_n)$ , their corresponding rewards  $\mathbf{r} = (r_1, r_2, \dots, r_n)$ , and the update size  $m$ . It outputs a subset of indices  $S \subseteq \{1, 2, \dots, n\}$ , where  $|S| = m$ , indicating which rollouts to retain for the policy update phase.

Given a selected subset of indices  $S$ , we compute the advantage estimates using only the selected rollouts:  $a_{S,i} = (r_i - \mu_S) / \sigma_S$ , where  $\mu_S$  and  $\sigma_S$  are the mean and standard deviation of the rewards in the selected subset. The GRPO-PODS objective then becomes:

$$L_{\text{PODS}}(\theta, S) = \frac{1}{m} \sum_{i \in S} \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \min \left[ \frac{\pi_{\theta}(o_{i,t} \mid p, o_{i,<t})}{\pi_{\theta_{\text{fixed}}}(o_{i,t} \mid p, o_{i,<t})} \cdot a_{S,i}, \text{clip} \left( \frac{\pi_{\theta}(o_{i,t} \mid p, o_{i,<t})}{\pi_{\theta_{\text{fixed}}}(o_{i,t} \mid p, o_{i,<t})}, 1 - \varepsilon, 1 + \varepsilon \right) \cdot a_{S,i} \right].$$

Our framework can be summarized as follows:

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#### Algorithm 1 The PODS Framework for GRPO

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**Input:** Models  $\pi_{\theta}, \pi_{\theta_{\text{fixed}}}$ , input prompt  $p$ , reward model  $R$ ,

Number of rollouts  $n$ , update size  $m$ , down-sampling rule  $D$

- 1: Independently sample  $n$  rollouts  $\mathbf{o} = (o_1, o_2, \dots, o_n)$  using  $\pi_{\theta_{\text{fixed}}}$  for prompt  $p$
- 2: Compute rewards  $\mathbf{r} = (r_1, r_2, \dots, r_n)$  using the reward model  $R$
- 3: Down-sample a set of  $m$  rollouts  $S \leftarrow D(\mathbf{o}, \mathbf{r}; m)$
- 4: Update the policy using the GRPO-PODS objective  $L_{\text{PODS}}(\theta, S)$

**Output:** An updated model  $\pi_{\theta_{\text{updated}}}$

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Algorithm 1 outlines the PODS framework for GRPO with a single prompt  $p$  in a training iteration. When training on a batch of multiple prompts, we simply apply the same procedure to each prompt and then concatenate the down-sampled rollouts and rewards. We conclude this section by presenting two intuitive down-sampling strategies that can potentially be plugged into PODS.

**Random down-sampling.** The rule  $D_{\text{rand}}$  uniformly selects  $m$  indices from  $\{1, 2, \dots, n\}$  without replacement, thereby preserving the statistical properties of the original rollout distribution. In expectation, it yields the same parameter update as running standard GRPO on exactly  $m$  rollouts.

**Max-reward down-sampling.** The rule  $D_{\text{maxr}}$  selects the  $m$  rollouts with the highest rewards, concentrating on examples that exhibit the most desirable behavior. This should allow the model to learn primarily from successful reasoning patterns. However, as we show in Section 4, ignoring low-reward rollouts deprives the policy of negative feedback and can significantly degrade performance.

### 3.3 Max-Variance Down-Sampling

We now introduce *max-variance down-sampling*, an information-theoretically motivated rule that selects the most diverse and informative rollouts according to their reward distribution.

Specifically,  $D_{\text{maxv}}$  chooses the subset  $S$  of size  $m$  that maximizes the empirical reward variance, i.e.,  $S = \arg \max_{|S|=m} \text{Var}(\{r_i \mid i \in S\})$ . By spanning the full performance spectrum, it supplies strong contrastive signals between successful and unsuccessful reasoning paths. Recent work by Razin and Krelov [38] provides an optimization-theoretic justification for this criterion.

A naive search would examine  $O(\binom{n}{m})$  subsets. This is clearly infeasible for realistic  $n$  and  $m$ . We prove, however, that the optimal subset can be found in  $O(n \log n)$  time.

**Lemma 3.1.** For a sorted list of rewards  $r_1 \leq r_2 \leq \dots \leq r_n$ , the variance-maximizing subset of size  $m$  always consists of the  $k$  highest rewards and  $(m - k)$  lowest rewards for some  $k \in \{0, 1, \dots, m\}$ . That is,

$$\text{Var}(\{r_1, \dots, r_{m-k}\} \cup \{r_{n-k+1}, \dots, r_n\}) = \max_{|S|=m} \text{Var}(\{r_i \mid i \in S\}).$$

**Proof of Lemma 3.1:** Let  $S^* = \arg \max_{|S|=m} \text{Var}(\{r_i \mid i \in S\})$  be the optimal subset of size  $m$ . We will show that if  $S^*$  is not of the form  $\{1, \dots, m-k\} \cup \{n-k+1, \dots, n\}$  for any  $k$ , then we can modify  $S^*$  to obtain a new subset  $S'$  of the same size with no smaller variance in rewards. By repeating this procedure, we can eventually reach a subset of this form.

Let  $\mu$  be the mean of the rewards in  $S^*$ . Since the set  $S^*$  does not take the form of  $\{1, \dots, m-k\} \cup \{n-k+1, \dots, n\}$  for any  $k$ , there exists either (i) an element  $i \in S^*$  such that  $i > 1, r_i \leq \mu$  and  $i-1 \notin S^*$ , or (ii) an element  $j \in S^*$  such that  $j < n, r_j \geq \mu$  and  $j+1 \notin S^*$ . That is, there exists an element in  $S^*$ , such that another element further from  $\mu$  is not in  $S^*$ . We will show that we can swap them without decreasing variance.

For the ease of notation, we will denote  $\text{Var}(\{r_i \mid i \in S\})$  as  $\text{Var}(S)$  in this proof.

For case (i), let  $S' = (S^* \setminus \{i\}) \cup \{i-1\}$ , and let  $\mu'$  be the mean of the rewards in  $S'$ . Then

$$\begin{aligned} \text{Var}(S') - \text{Var}(S^*) &= \left( \frac{1}{m} \sum_{t \in S'} r_t^2 - \mu'^2 \right) - \left( \frac{1}{m} \sum_{t \in S^*} r_t^2 - \mu^2 \right) \\ &= \frac{1}{m} (r_{i-1}^2 - r_i^2) - (\mu'^2 - \mu^2) \\ &= \frac{1}{m} (r_{i-1} - r_i)(r_{i-1} + r_i) - (\mu' - \mu)(\mu' + \mu) \\ &= \frac{1}{m} (r_{i-1} - r_i)[(r_{i-1} + r_i) - (\mu' + \mu)] \geq 0. \end{aligned}$$

For case (ii), let  $S' = (S^* \setminus \{j\}) \cup \{j+1\}$ , we can similarly show that  $\text{Var}(S') - \text{Var}(S^*) \geq 0$ .

In either case, we have shown that we can modify  $S^*$  to obtain a new subset  $S'$  of the same size that has no smaller variance in rewards. We can repeat this process until we reach a subset of the form  $\{1, \dots, m-k\} \cup \{n-k+1, \dots, n\}$  for some  $k$ . Thus, we conclude that there must exist one optimal subset of this form for some  $k$ . ■

Lemma 3.1 naturally leads to a practical algorithm, Algorithm 2, for max-variance down-sampling. Moreover, it also offers intuition as to why maximizing variance is effective: the optimal subset contains the  $k$  highest rewards and the  $(m - k)$  lowest rewards, thereby capturing strong contrastive signals from both positive and negative examples.

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#### Algorithm 2 Max-Variance Down-Sampling

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**Input:** Number of rollouts  $n$ , update size  $m$ , rollouts  $\{o_1, o_2, \dots, o_n\}$ , rewards  $\{r_1, r_2, \dots, r_n\}$   
1: Sort the rollouts by reward and get the sorted indices  $ind \leftarrow \text{argsort}(\{r_1, r_2, \dots, r_n\})$   
2: Let  $S_{\text{ans}} \leftarrow \{ind_1, \dots, ind_m\}$   
3: **for**  $k \in \{1, \dots, m\}$  **do**  
4:   Let  $S_{\text{this}} \leftarrow \{ind_1, \dots, ind_{m-k}\} \cup \{ind_{n-k+1}, \dots, ind_n\}$   
5:   Let  $S_{\text{ans}} \leftarrow S_{\text{this}}$  **if**  $\text{Var}(\{r_i \mid i \in S_{\text{this}}\}) > \text{Var}(\{r_i \mid i \in S_{\text{ans}}\})$   
6: **end for**  
**Output:** Selected indices  $S_{\text{ans}}$  of rollouts

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**Theorem 1.** Algorithm 2 computes the max-variance down-sampling rule correctly. Moreover, it can be implemented in  $O(n \log n)$  time.

**Proof of Theorem 1:** The correctness of Algorithm 2 follows directly from Lemma 3.1.

For the time complexity, we first sort the rewards in  $O(n \log n)$  time. To compute the variance of the selected rollouts, note that  $\text{Var}(\{x \mid x \in S_{\text{this}}\}) = \mathbf{E}_{x \in S_{\text{this}}} [x^2] - (\mathbf{E}_{x \in S_{\text{this}}} [x])^2$ . We can maintain the prefix sums of the rewards and the squared rewards in  $O(n)$  time. Then, for each  $k$ , we can

compute the variance of the selected rollouts in  $O(1)$  time using the prefix sums. Thus, the overall time complexity is  $O(n \log n) + O(m) = O(n \log n)$ . ■

Theorem 1 shows that the max-variance down-sampling rule can be computed efficiently, which enables its practical application in GRPO-PODS. We conclude this section by noting an important special case of the max-variance down-sampling rule.

**Theorem 2.** *Let  $m$  be an even integer. When the rewards are binary, selecting  $m/2$  rollouts with the highest rewards and  $m/2$  rollouts with the lowest rewards maximizes the variance of the rewards.*

**Proof of Theorem 2:** Let the number of rollouts with reward 1 be  $k$ . Then, the number of rollouts with reward 0 is  $n - k$ . If  $k \leq m/2$ , then any subset of  $m$  rollouts contains at most  $k$  rollouts with reward 1, and the variance is maximized by selecting these  $k$  rollouts and any  $(m - k)$  rollouts with reward 0. If  $n - k \leq m/2$ , then any subset of  $m$  rollouts contains at most  $(n - k)$  rollouts with reward 0, and the variance is maximized by selecting these  $(n - k)$  rollouts and any  $m - (n - k)$  rollouts with reward 1. Otherwise, any subset of  $m/2$  rollouts with reward 1 and  $m/2$  rollouts with reward 0 maximizes the variance. In all cases, we can select  $m/2$  rollouts with the highest rewards and  $m/2$  rollouts with the lowest rewards to maximize the variance. This concludes the proof. ■

## 4 Experiments

We evaluate PODS on two reasoning benchmarks—GSM8K [7] and MATH [8], both released under the MIT licence—across two hardware and model regimes:

- (a) LoRA fine-tuning of Qwen2.5-3B-Instruct (rank 64,  $\alpha = 64$ ) on a single L40S GPU.
- (b) Full-parameter fine-tuning of the same model on eight H100 GPUs.

**Implementation details.** For regime (a), we use Unsloth [39] together with TRL [40]. The maximum sequence length is 1024. Optimization uses AdamW with learning rate  $5 \times 10^{-6}$ , weight decay 0.1, and gradient-norm clipping at 1.0. A rule-based reward model scores each rollout for correctness and format. Following [20], we omit the KL-divergence term from the GRPO objective because it is unnecessary for reasoning tasks. For regime (b), distributed training is implemented with DeepSpeed ZeRO-2 [41]. We extend the open-r1 library [42] to support PODS on multiple GPUs. We use the same hyperparameters as in regime (a), except we use a maximum sequence length of 1536. We adopt the same reward functions as open-r1 [42]. We publish the code for our experiments at <https://github.com/YixuanEvenXu/pods>.

**Section roadmap.** In Section 4.1, we compare the performance of GRPO and GRPO-PODS across different datasets and hardware environments. We show that for all the settings we test, GRPO-PODS consistently outperforms GRPO in terms of performance as the training time increases. Then, in Sections 4.2 and 4.3, we analyze the effect of the rollout and update sizes  $(n, m)$  and the down-sampling rules on the performance of GRPO-PODS, respectively. We provide empirical insights into how to choose the rollout and update sizes for GRPO-PODS, and show that the max-variance down-sampling rule consistently outperforms other down-sampling rules we consider. We present some additional evaluation results of the models we obtain in Appendix A.

### 4.1 Comparing GRPO and GRPO-PODS

We compare the performance of GRPO and GRPO-PODS with max-variance down-sampling on three different settings of dataset and hardware environments (a) training on GSM8K with one L40S GPU, (b) training on MATH with one L40S GPU, and (c) training on GSM8K with 8 H100 GPUs.

The results are shown in Fig. 2. For setting (a), we set  $n = 16$  for GRPO and  $(n, m) = (64, 16)$  for GRPO-PODS. For setting (b), we set  $n = 8$  for GRPO and  $(n, m) = (32, 8)$  for GRPO-PODS due to the longer sequence length of MATH. In both cases, we set the gradient accumulation steps to 1.

For setting (c), we configure GRPO with  $n = 32$  and gradient accumulation steps = 16, processing one prompt per GPU. Gradient accumulation allows us to simulate larger batch sizes by accumulating gradients across multiple forward-backward passes before performing a parameter update, generating  $32 \times 16 = 512$  rollouts per prompt. For GRPO-PODS, we set  $(n, m) = (128, 32)$  with gradient

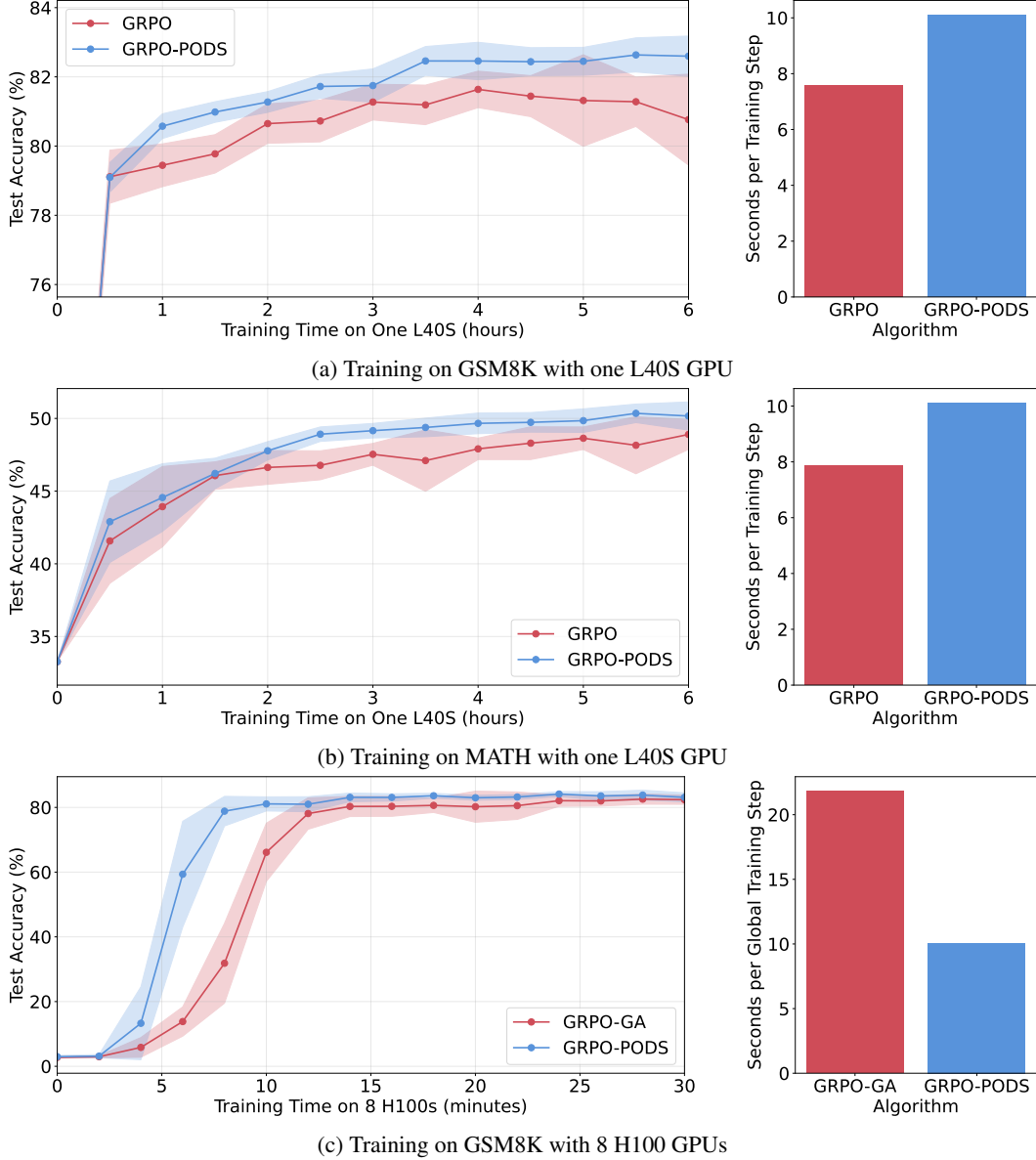


Figure 2: Performance and per-step run time comparison of standard GRPO and GRPO-PODS with max-variance down-sampling across different datasets and hardware environments. For the performance comparison, the x-axis shows the training time, and the y-axis shows the accuracy on the test set. The shaded area represents 1.96 times the standard error of the mean.

accumulation steps = 4, maintaining the same total of 512 rollouts per prompt while applying a downsampling ratio of 4. This configuration ensures that the total number of generated rollouts remains constant across both methods, isolating the effect of our down-sampling approach while maximizing inference utilization. We evaluate the model on the test set every 0.5 hours for setting (a) and (b), and every 2 minutes for setting (c). We report the average accuracy for each evaluation step.

We observe that GRPO-PODS consistently outperforms GRPO across all settings. In particular, GRPO-PODS converges to a higher accuracy than GRPO in a shorter amount of time, regardless of whether gradient accumulation is used. This shows that our proposed down-sampling method is effective in improving the performance of GRPO on reasoning tasks like GSM8K and MATH.

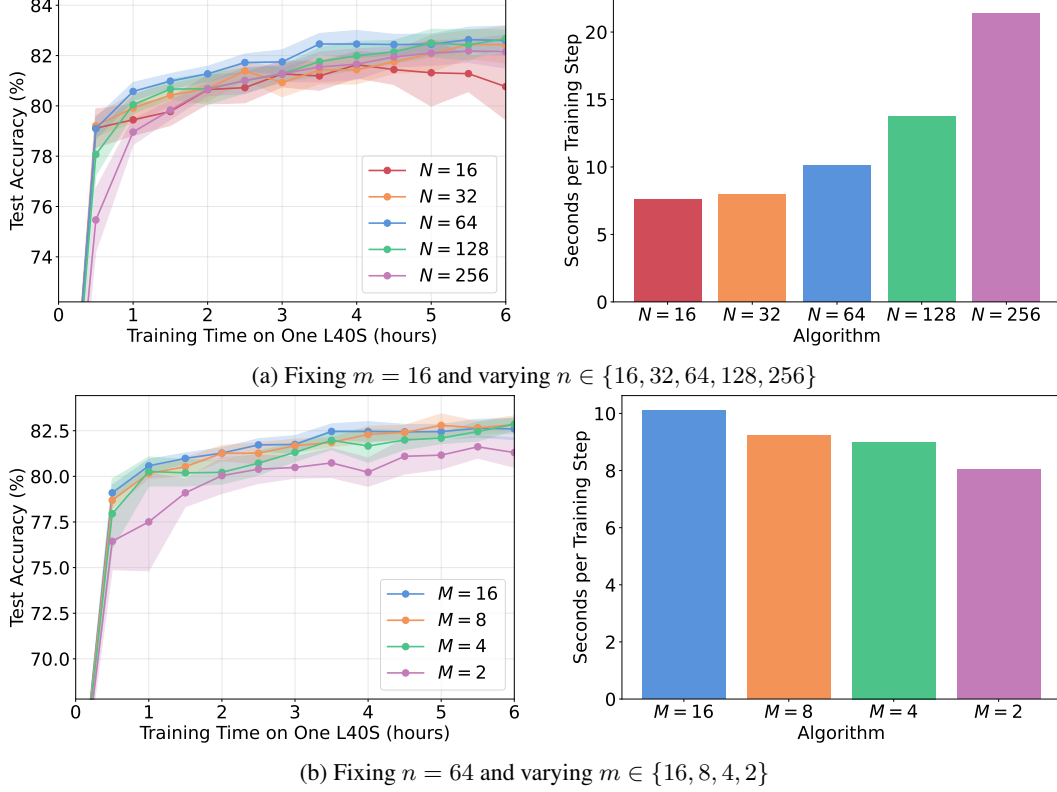


Figure 3: Performance and per-step run time comparison of GRPO-PODS with max-variance down-sampling across different settings of  $n$  and  $m$ . The training is conducted on the GSM8K dataset with one L40S. For the performance comparison, the x-axis shows the training time, and the y-axis shows the accuracy on the test set. The shaded area represents 1.96 times the standard error of the mean.

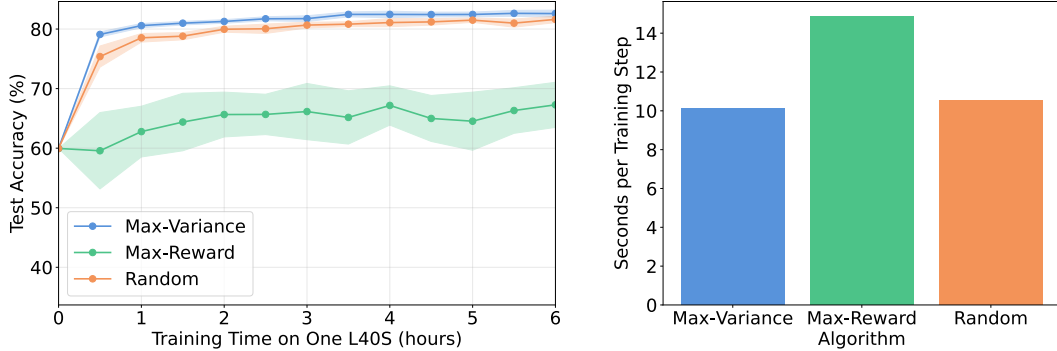


Figure 4: Performance and per-step run time comparison of GRPO-PODS with the max-variance, max-reward and random down-sampling rules. The training is conducted on the GSM8K dataset with one L40S. For the performance comparison, the x-axis shows the training time, and the y-axis shows the accuracy on the test set. The shaded area represents 1.96 times the standard error of the mean.

#### 4.2 Effect of Rollout and Update Sizes ( $n, m$ )

We study the effect of the rollout size  $n$  and update size  $m$  on the performance of GRPO-PODS with max-variance down-sampling in this section. We conduct experiments on the GSM8K dataset with one L40S GPU, and we vary  $n$  and  $m$  independently. The results are shown in Fig. 3. For the rollout size scaling experiment, we fix  $m = 16$  and vary  $n \in \{16, 32, 64, 128, 256\}$ . For the update size scaling experiment, we fix  $n = 64$  and vary  $m \in \{16, 8, 4, 2\}$ .



We observe that increasing the rollout size  $n$  results in a single-peaked performance curve, where the performance first increases and then decreases as  $n$  increases. This can be attributed to two factors: (1) as shown in the right plot of Fig. 3a, the per-step run time increases as  $n$  increases, and such increase is more pronounced when  $n$  is large enough to saturate the GPU memory; (2) the overall quality of the rollouts retained for training increases as  $n$  increases, but the marginal gain diminishes. Overall, we find that  $n = 64$  is a good choice for the rollout size when  $m = 16$ .

For the update size scaling experiment, we observe that decreasing  $m$  results in an increased variance of the algorithm’s performance, but the overall performance is not significantly affected unless we decrease  $m$  to a very small value like 2 or 4. This indicates that GRPO-PODS with max-variance down-sampling is robust to the choice of  $m$  as long as it is not too small.

### 4.3 Comparing Different Down-Sampling Rules

We study the effect of different down-sampling rules on the performance of GRPO-PODS in this section. We conduct experiments on the GSM8K dataset with one L40S GPU. We set the rollout size  $n = 64$  and the update size  $m = 16$ , and we compare three different down-sampling rules: (1) max-variance down-sampling, (2) max-reward down-sampling, and (3) random down-sampling. The results are shown in Fig. 4. We observe that the max-variance down-sampling rule consistently outperforms both the max-reward and random down-sampling rules across all settings. This indicates that the max-variance down-sampling rule is effective in selecting informative rollouts for training.

## 5 Conclusion and Discussion

We introduced **PODS**—a lightweight, algorithm-agnostic framework that tackles the inference-update asymmetry in RL for LLMs. PODS generates large batches of rollouts in parallel and updates the policy on only an informative subset chosen by the max-variance rule. The method preserves the embarrassingly parallel scalability of inference while lowering the communication and memory costs of training. Our analysis shows that the optimal subset can be found in  $O(n \log n)$  time.

Across multiple datasets and hardware configurations, GRPO-PODS consistently outperforms standard GRPO under equal wall-clock budgets, converging faster and reaching higher final accuracy. Ablation studies yield two main insights: (i) performance is robust over a wide range of down-sampling ratios provided  $m$  is not too small, and (ii) variance-based selection clearly surpasses random and reward-only baselines, empirically confirming our theoretical motivation.

**Limitations.** Our evaluation focuses on mathematical-reasoning tasks with verifiable rewards. Other domains such as open-ended dialogue or code generation may exhibit distinct dynamics of the algorithms. Moreover, in workloads that demand greater prompt diversity, similar gains might be obtained by processing more prompts with fewer rollouts per prompt and accumulating gradients across prompts—an alternative path to address the inference-update asymmetry. Finally, because PODS alters the training rollout distribution through selective down-sampling, it behaves off-policy and may be unsuitable when strict on-policy guarantees are required.

**Future work.** The algorithm-agnostic nature of PODS enables integration with value-based methods like PPO and emerging RL approaches. Exploring whether PODS can enhance state-of-the-art RL pipelines represents a promising research direction. Additionally, investigating adaptive down-sampling strategies that evolve throughout training could further optimize the learning dynamics. Exploring theoretically principled approaches to balance the trade-off between prompt diversity and rollout quantity per prompt also warrants investigation.

**Broader impact.** We anticipate our work will primarily have positive social impact by improving the computational efficiency and effectiveness of RL training for LLMs, potentially democratizing access to high-quality reasoning models. However, by lowering the computational barriers to training powerful reasoning systems, our method may accelerate capabilities that could be misused. This heightens the importance of responsible release practices to mitigate harmful behaviors. Our open-source release of code and experimental frameworks aims to facilitate reproducibility while encouraging informed and safe adoption within the research community.

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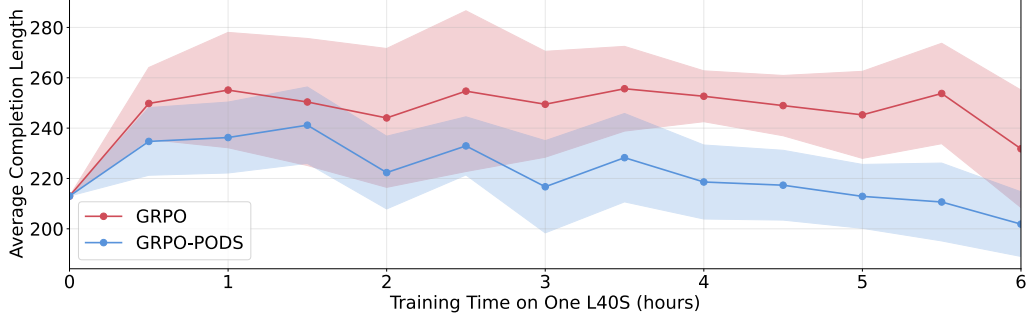
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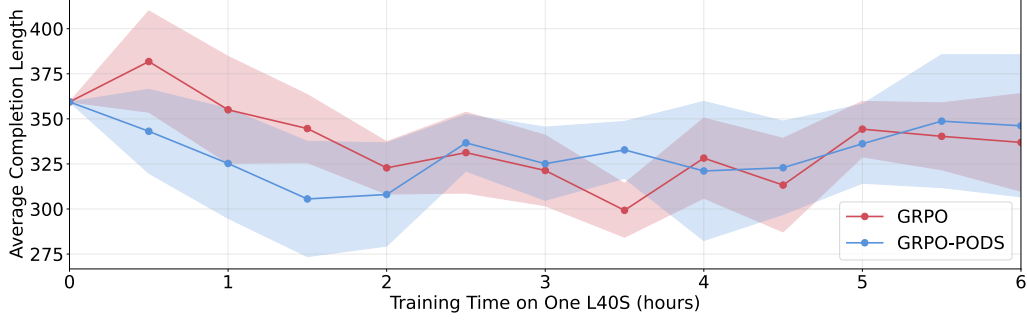
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## A Additional Experimental Details

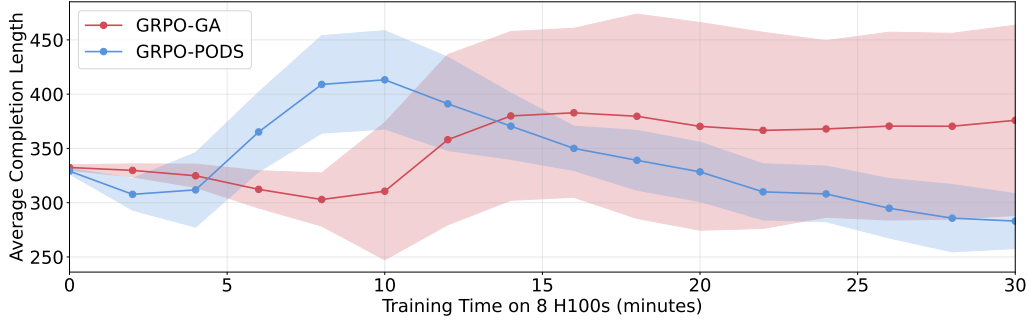
We include additional evaluation of the average completion length over the training time for each of the experiments we conduct in Section 4. We present the average completion length results in Figs. 5 to 7, in correspondence to Figs. 2 to 4 respectively. In most of the cases, we observe that the average completion length stays relatively stable over the training time.



(a) Training on GSM8K with one L40S GPU

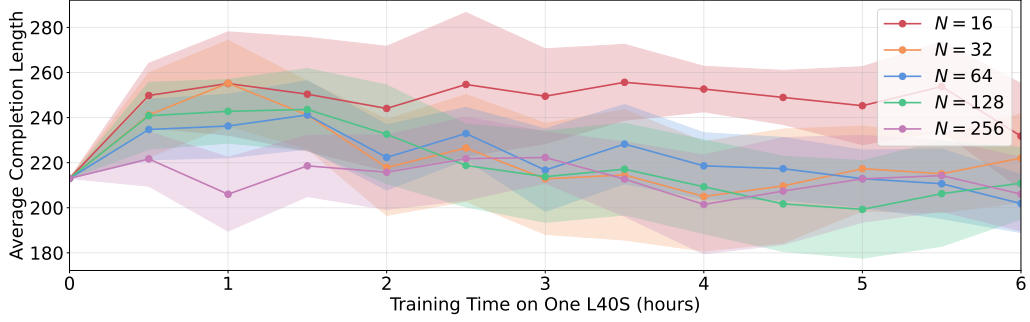


(b) Training on MATH with one L40S GPU

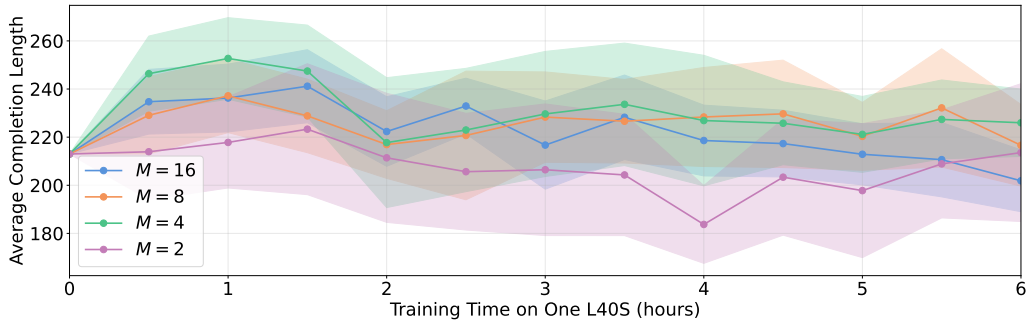


(c) Training on GSM8K with 8 H100 GPUs

Figure 5: Average completion length over time of the trained models in Section 4.1’s experiments. The x-axis shows the training time, and the y-axis shows the average completion length in tokens. The shaded area represents 1.96 times the standard error of the mean.



(a) Fixing  $m = 16$  and varying  $n \in \{16, 32, 64, 128, 256\}$



(b) Fixing  $n = 64$  and varying  $m \in \{16, 8, 4, 2\}$

Figure 6: Average completion length over time of the trained models in Section 4.2’s experiments. The x-axis shows the training time, and the y-axis shows the average completion length in tokens. The shaded area represents 1.96 times the standard error of the mean.

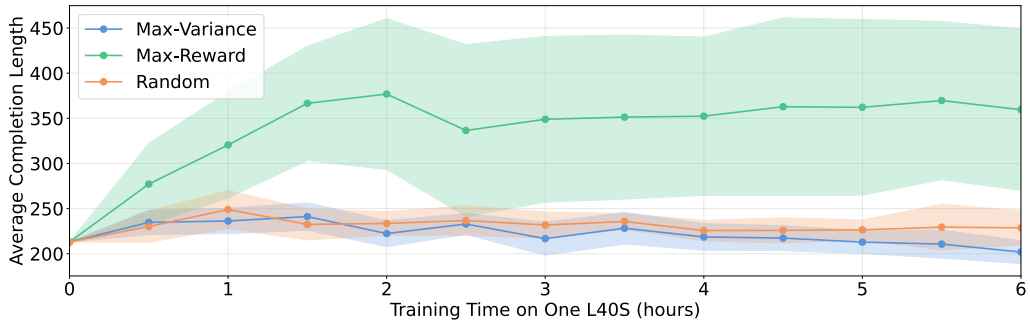


Figure 7: Average completion length over time of the trained models in Section 4.3’s experiments. The x-axis shows the training time, and the y-axis shows the average completion length in tokens. The shaded area represents 1.96 times the standard error of the mean.