Deviate or Not: Learning Coalition Structures with Multiple-bit Observations in Games

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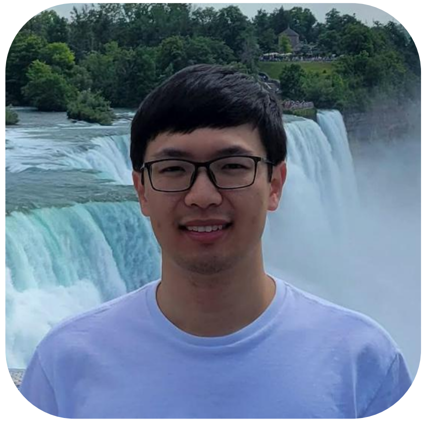

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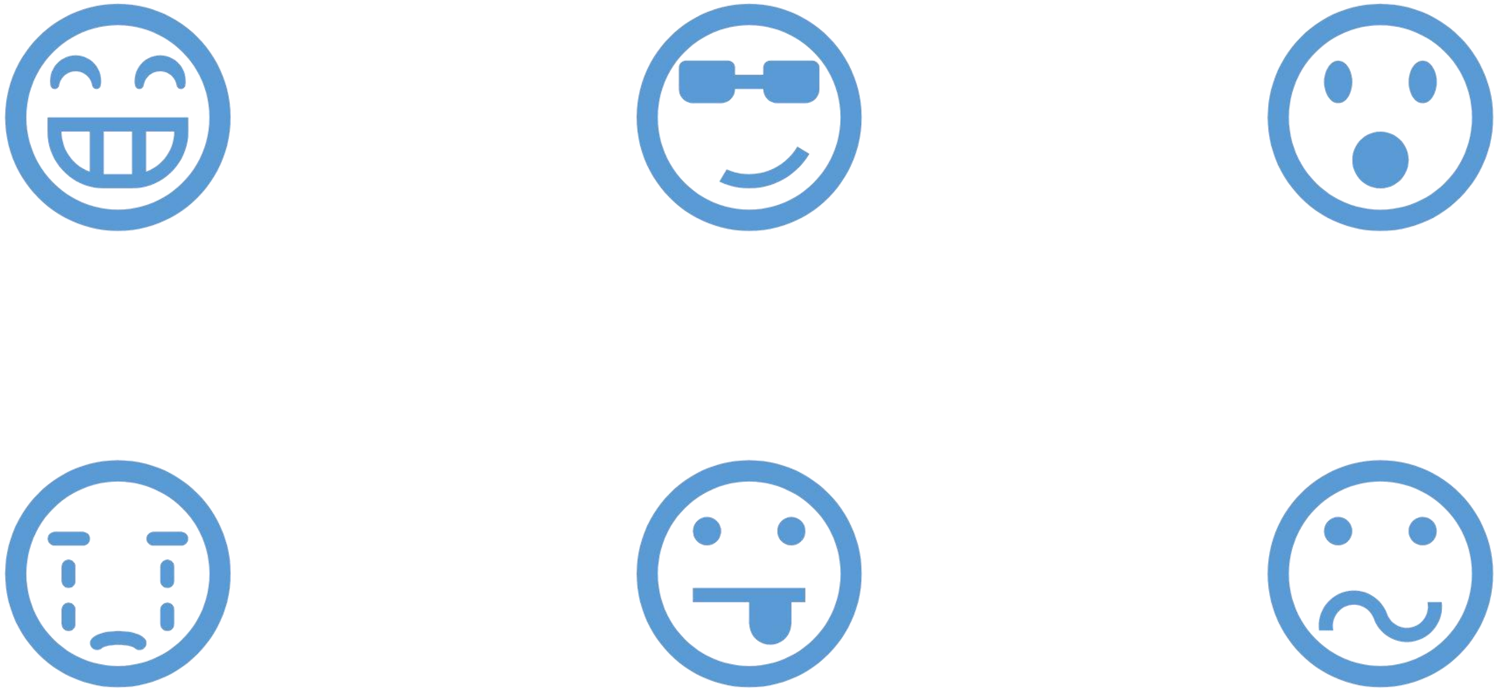
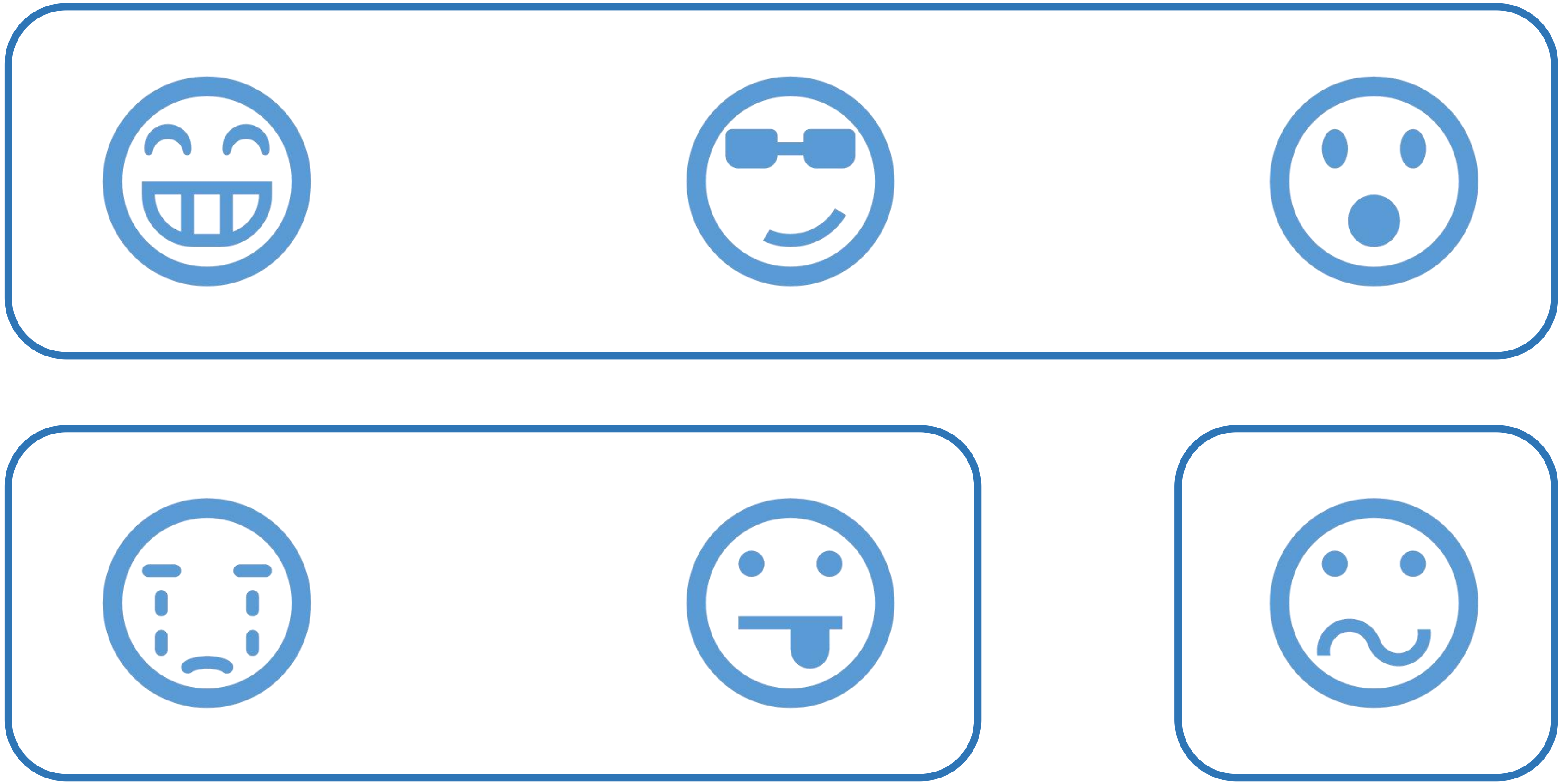
Coalition Structures

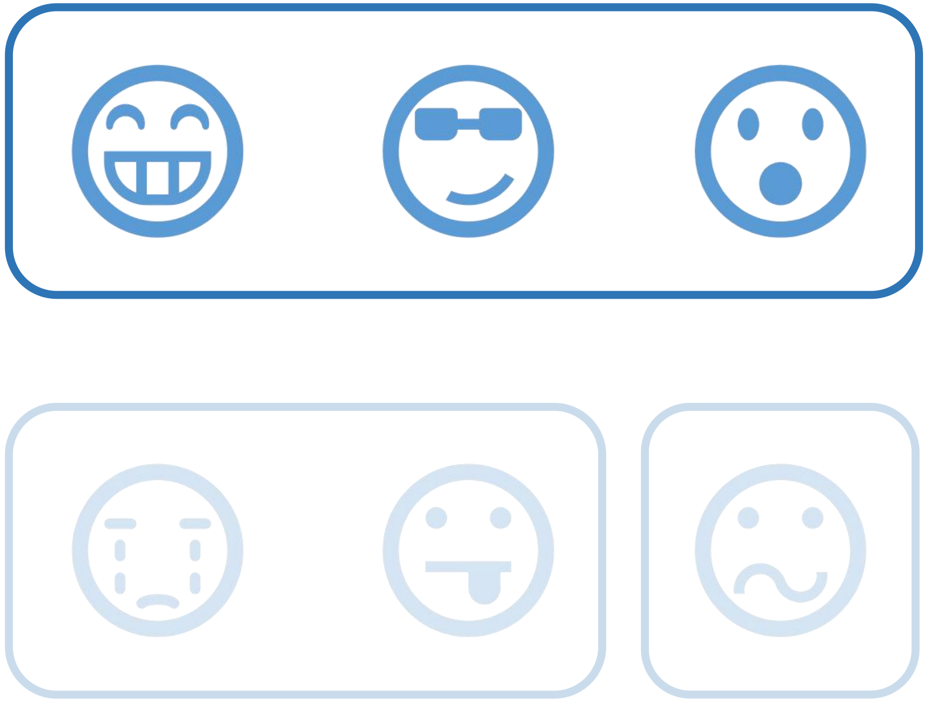


Coalition Structures



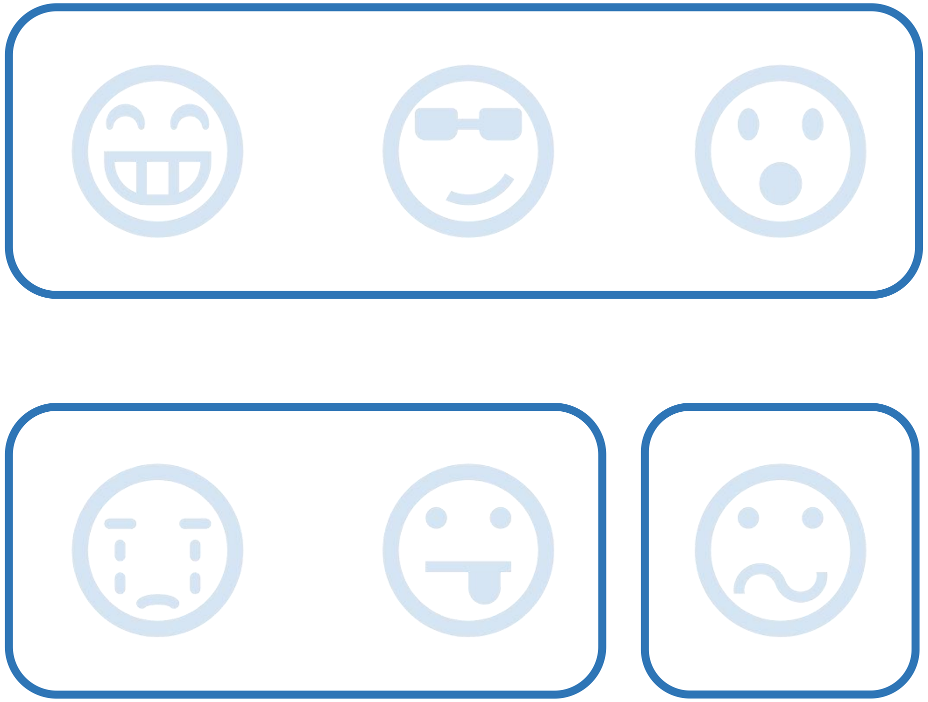
Coalition Structure Learning (CSL)

- **Coalition:** A nonempty subset of the agents, in which
  - The agents **coordinate their actions**
  - The agents **have common interests**



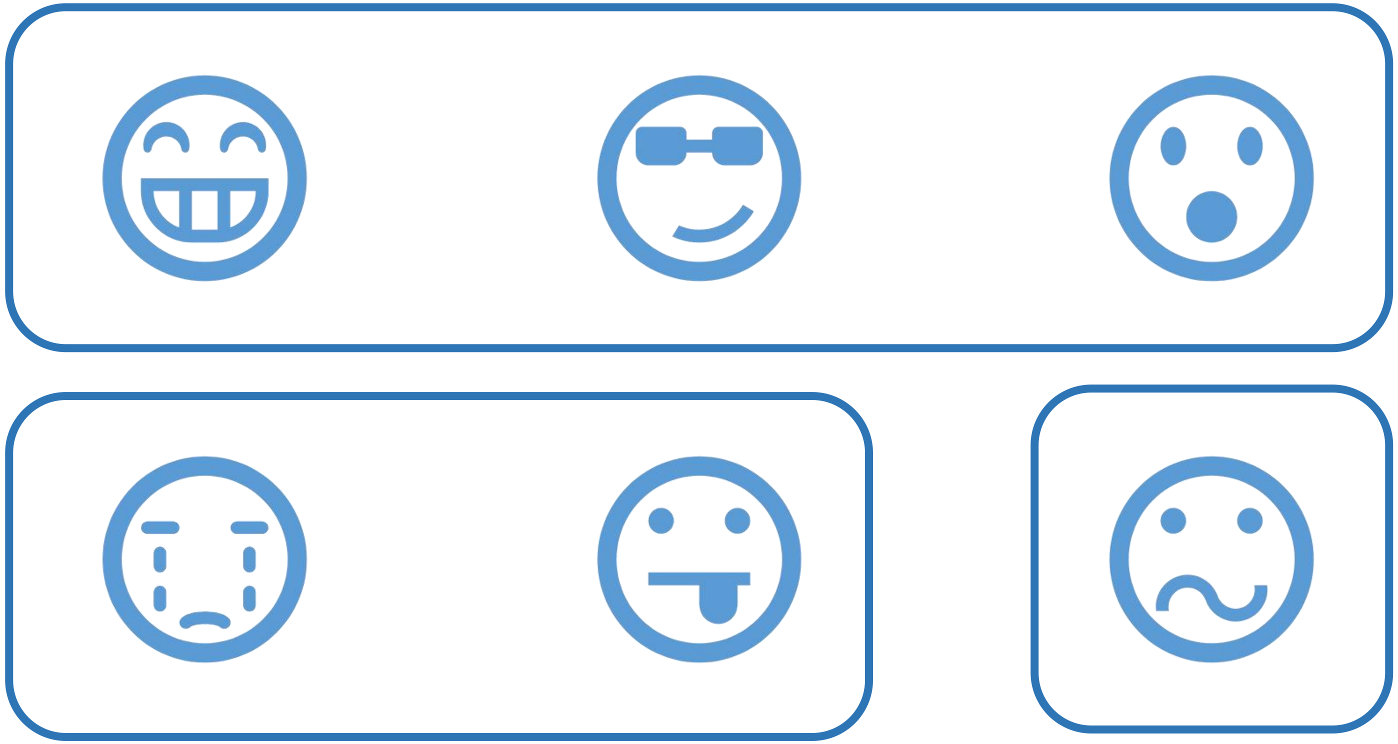
Coalition Structure Learning (CSL)

- **Coalition:** A nonempty subset of the agents, in which
  - The agents **coordinate their actions**
  - The agents **have common interests**
- **Coalition Structure:** A set partition of the agents $\{1,\ 2, \cdots, n\}$
  - Each set is a separate coalition
  - **Behavior Model in a Game:** Coalition **acts as a joint player** whose actual utility equals the **total utilities of its members**

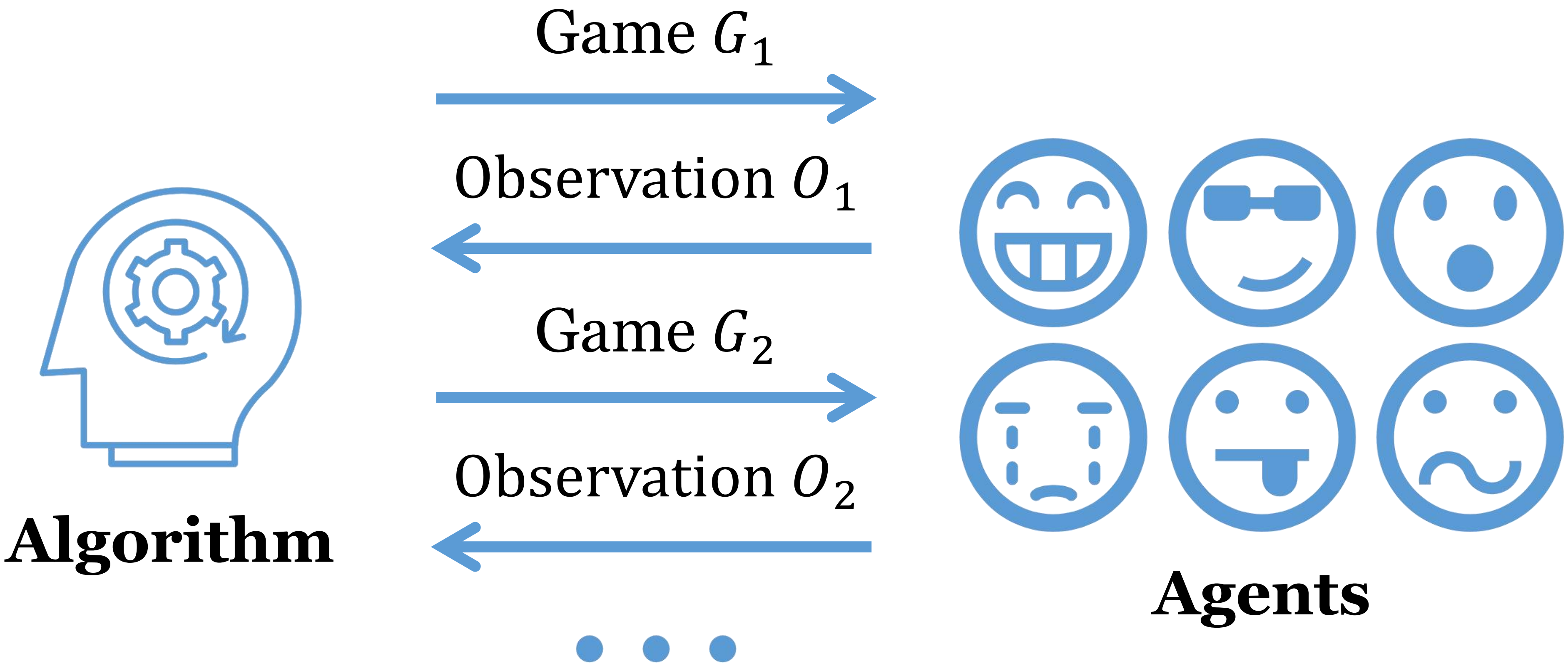


Coalition Structure Learning (CSL)

- **Coalition Structure Learning (CSL):** Recover the unknown coalition structure by observing interactions in designed games



Interactive Model



Single-Bit Observation (Xu et al. AAAI 2024)

- **Model:** The algorithm queries a game G and a strategy profile Σ, and the agents answer whether Σ is a **Nash Equilibrium** in G
  - Easy to compute for the agents
  - **One bit of information** per query

- **Theorem:** **Any algorithm** for CSL must interact **at least** $n \log_2 n - O(n \log_2 \log_2 n)$ rounds with the agents
  - We need this many bits of information to distinguish between answers
  - **Too large** for real-world systems with many agents

Multiple-Bit Observation (This Work)

- **Model:** The algorithm queries a game G and a strategy profile Σ, and each agent indicates **whether they want to deviate**
  - Still Easy to compute for the agents
  - ***n* bits of information** per query

- **Theorem:** Any algorithm for **Multiple-bit CSL** must interact at least $\log_2 n - O(\log_2 \log_2 n)$ rounds with the agents
  - This opens up the possibility of much more efficient algorithms

Types of Games

- What kind of games can the algorithm design?
  - Natural choice: **Normal-form games**
    - The **most general** one, thus the **easiest** for the algorithm
  - Succinct games: **Congestion games**, **graphical games**
  - More related to practice: **Auctions**

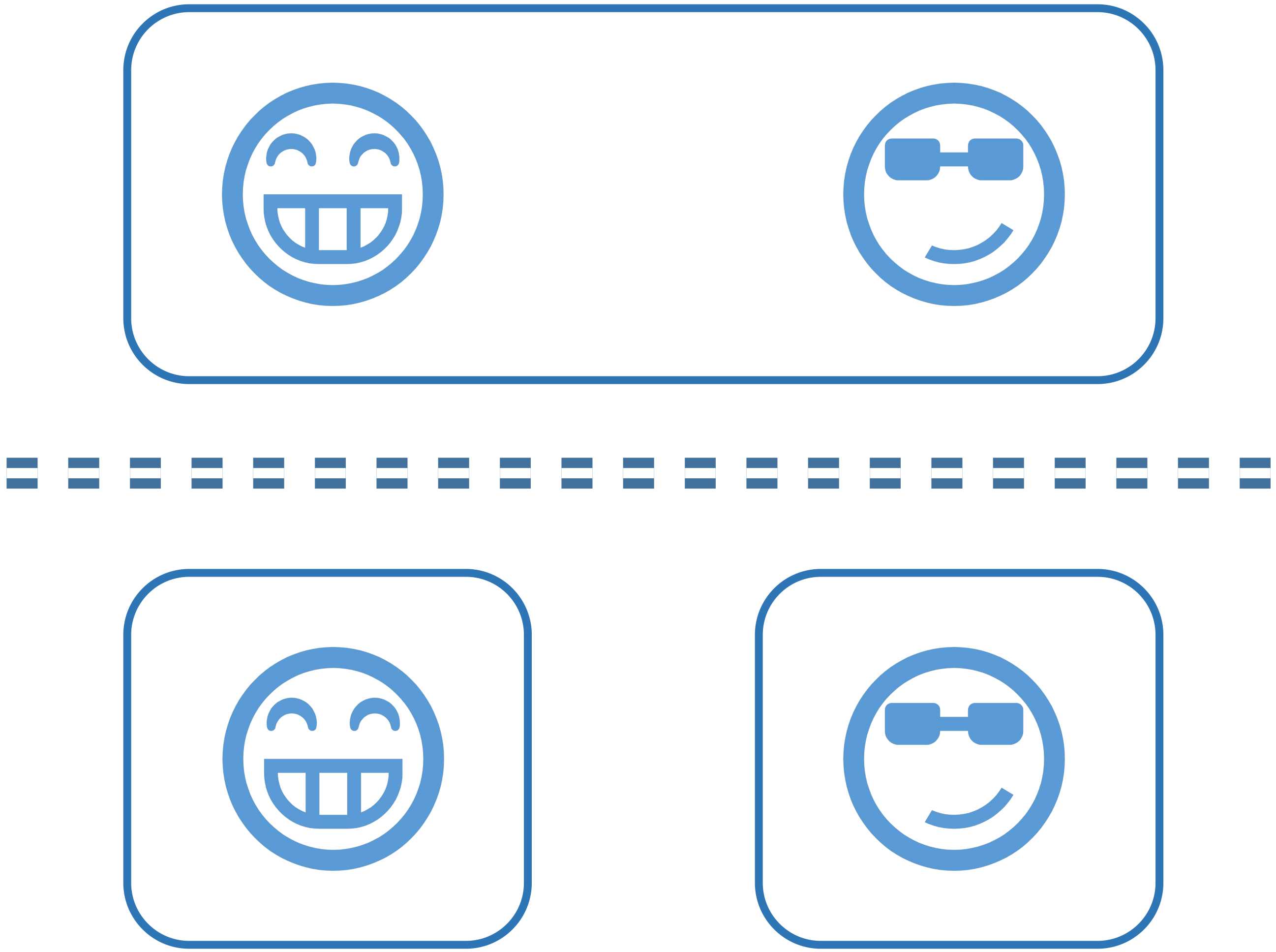
- We study **all** the above settings in this paper
  - And show **asymptotically optimal algorithms** for most of them
  - We **summarize** the results and the important techniques in the slides

Normal-form Games

- **Setting:** The algorithm may design any normal-form games
- **Lower bound:** $\log_2 n - O(\log_2 \log_2 n)$ rounds

- **Algorithm:** $\log_2 n + 2$ rounds in the worst case
  - **Gadget:** Product of **directed prisoner's dilemma**
  - **Key technique:** **Simultaneous binary search**
    - Find a representative in the coalition for each agent simultaneously
  - **Optimal** up to low-order terms

How to Distinguish Between the Two?



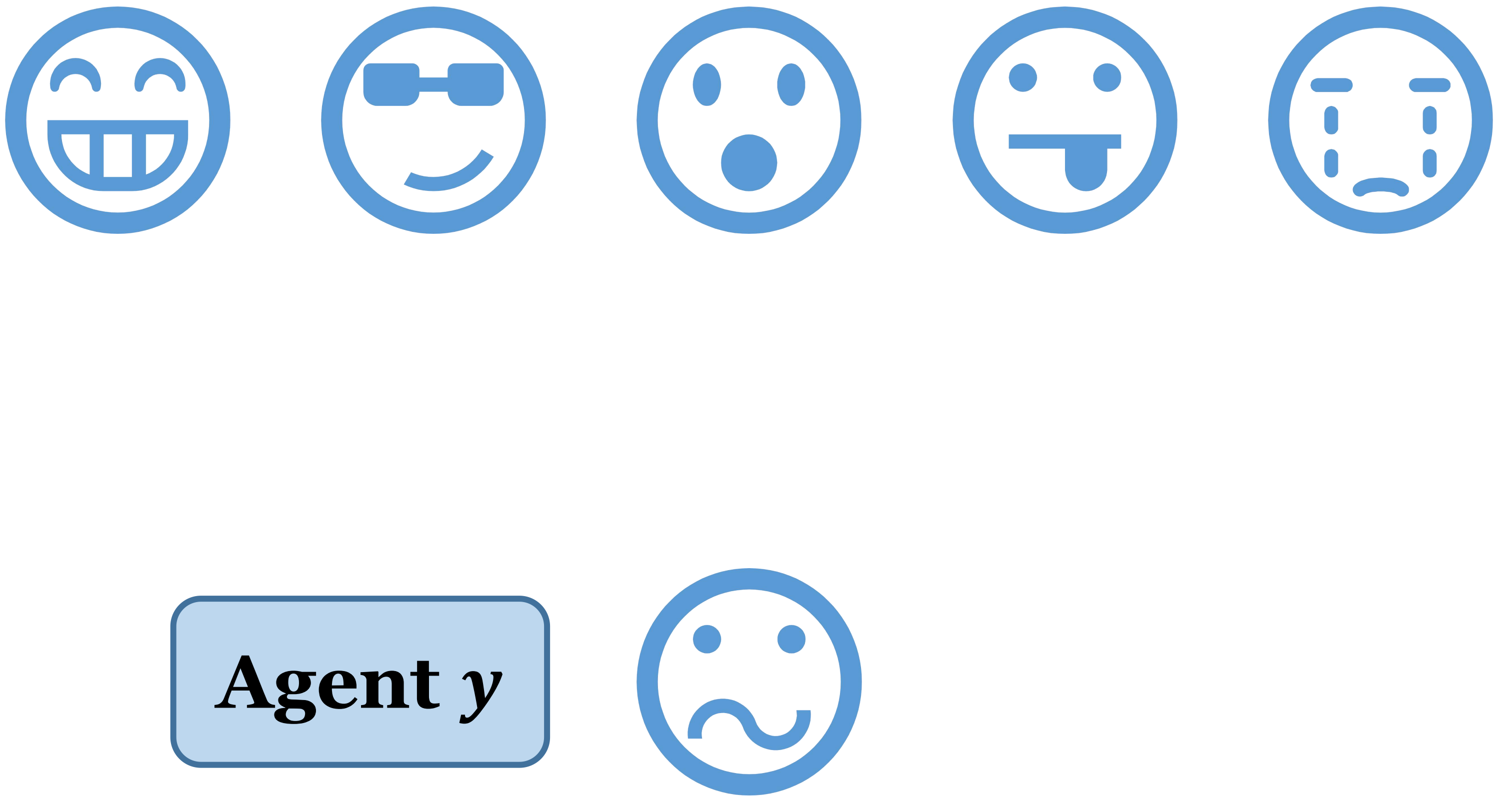
Directed Prisoner's Dilemma

- **Directed Prisoner's Dilemma:** A normal form game between agents (x, y), where agent y can choose cooperate, losing 1 unit of utility and giving agent x 2 units of utility

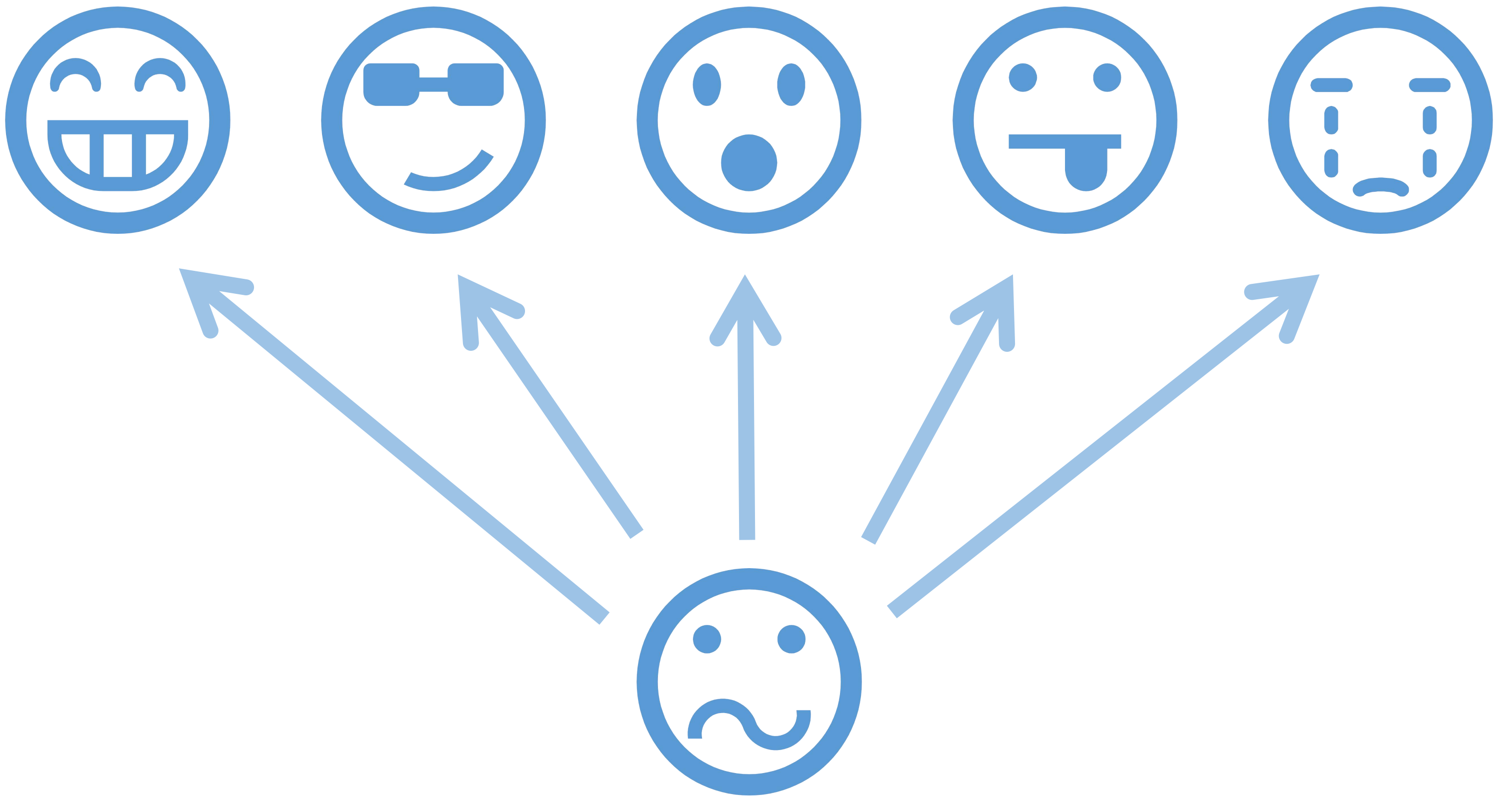
	$\boldsymbol{C_y}$	$\boldsymbol{D_y}$
D_x	(2, -1)	(0, 0)

- **Lemma 3.1:** Agent y **deviates** from D_y if and only if x and y are **in the same coalition**

Aggregated Observation



Aggregated Observation



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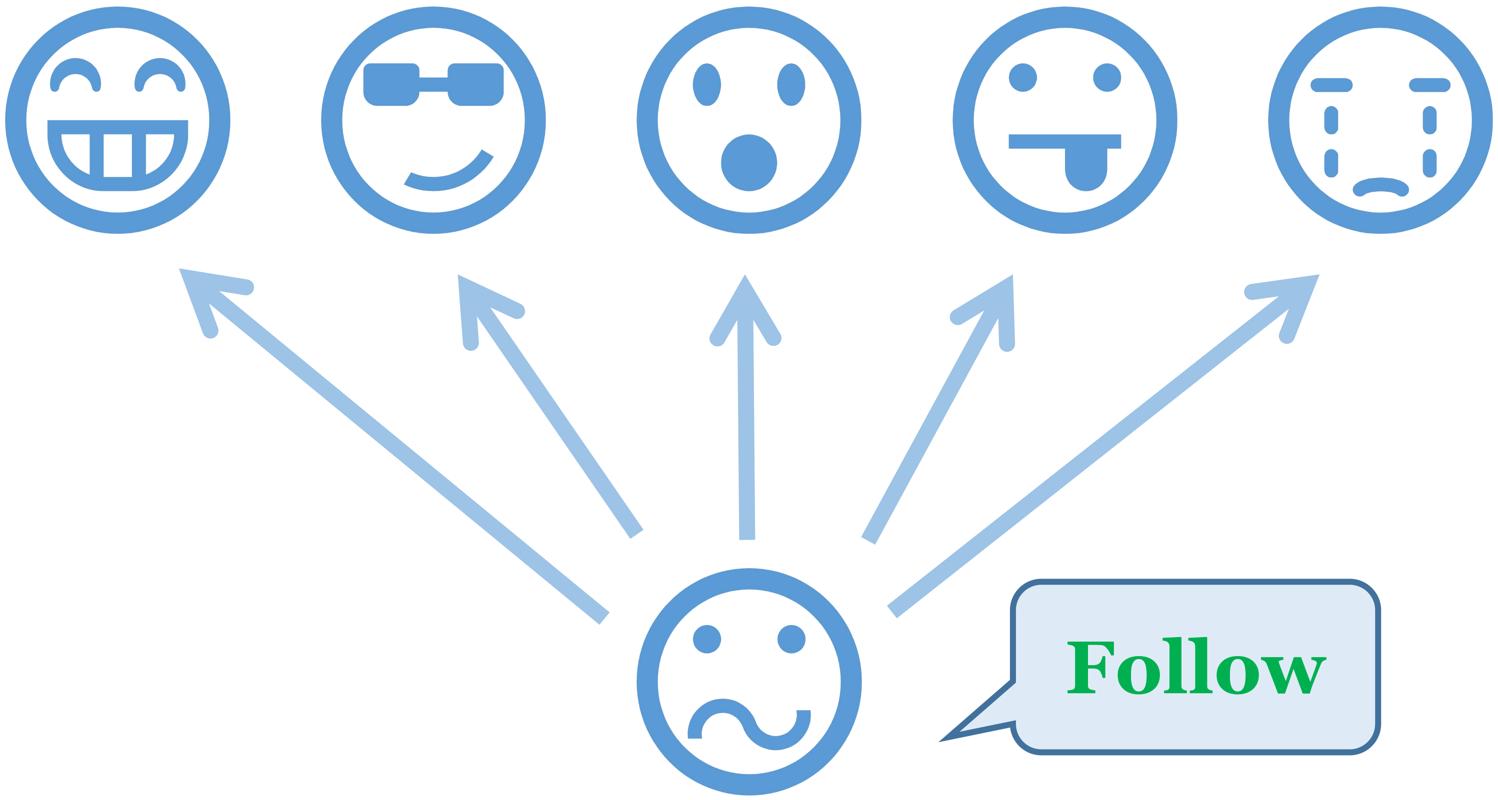

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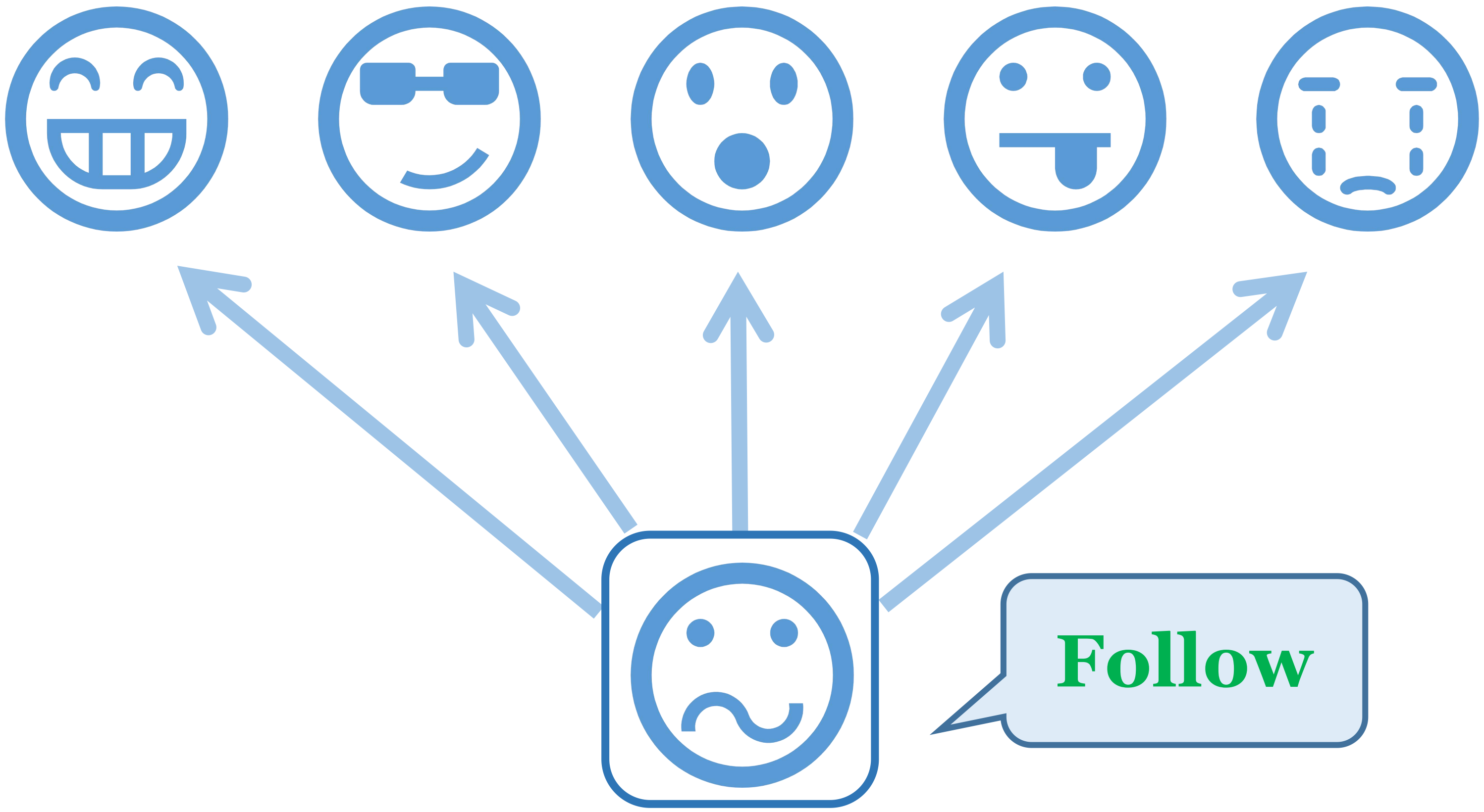

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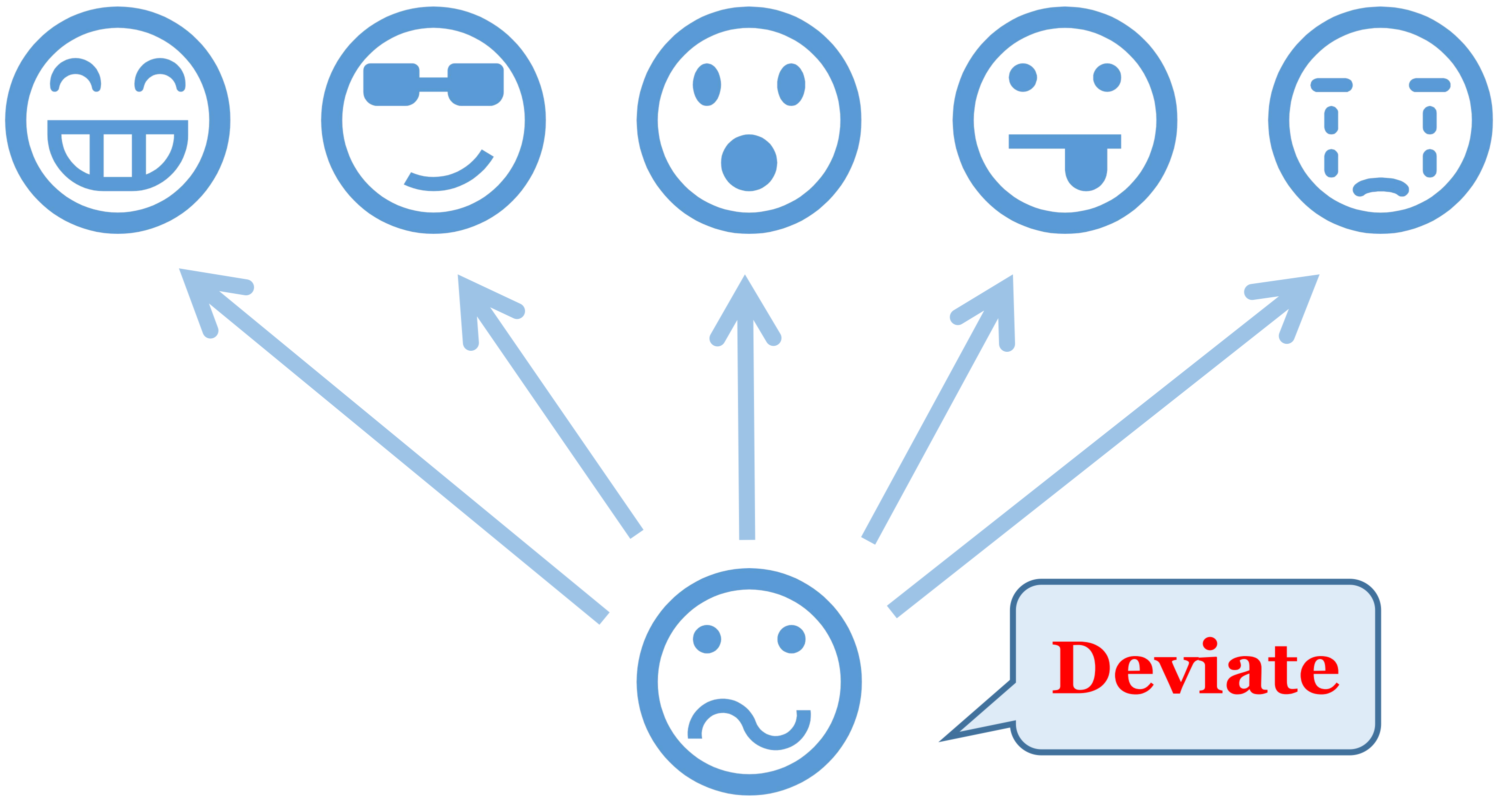
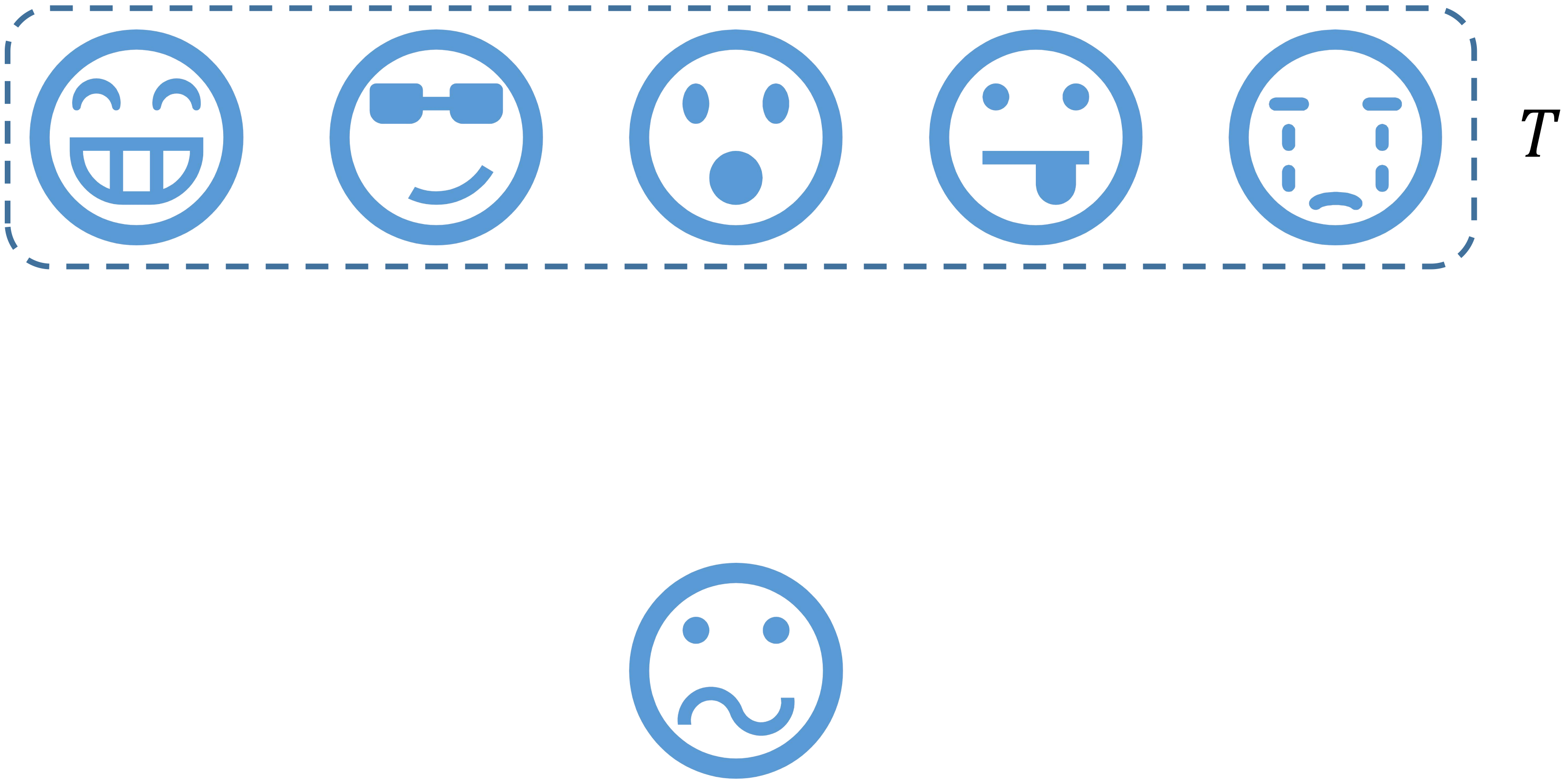
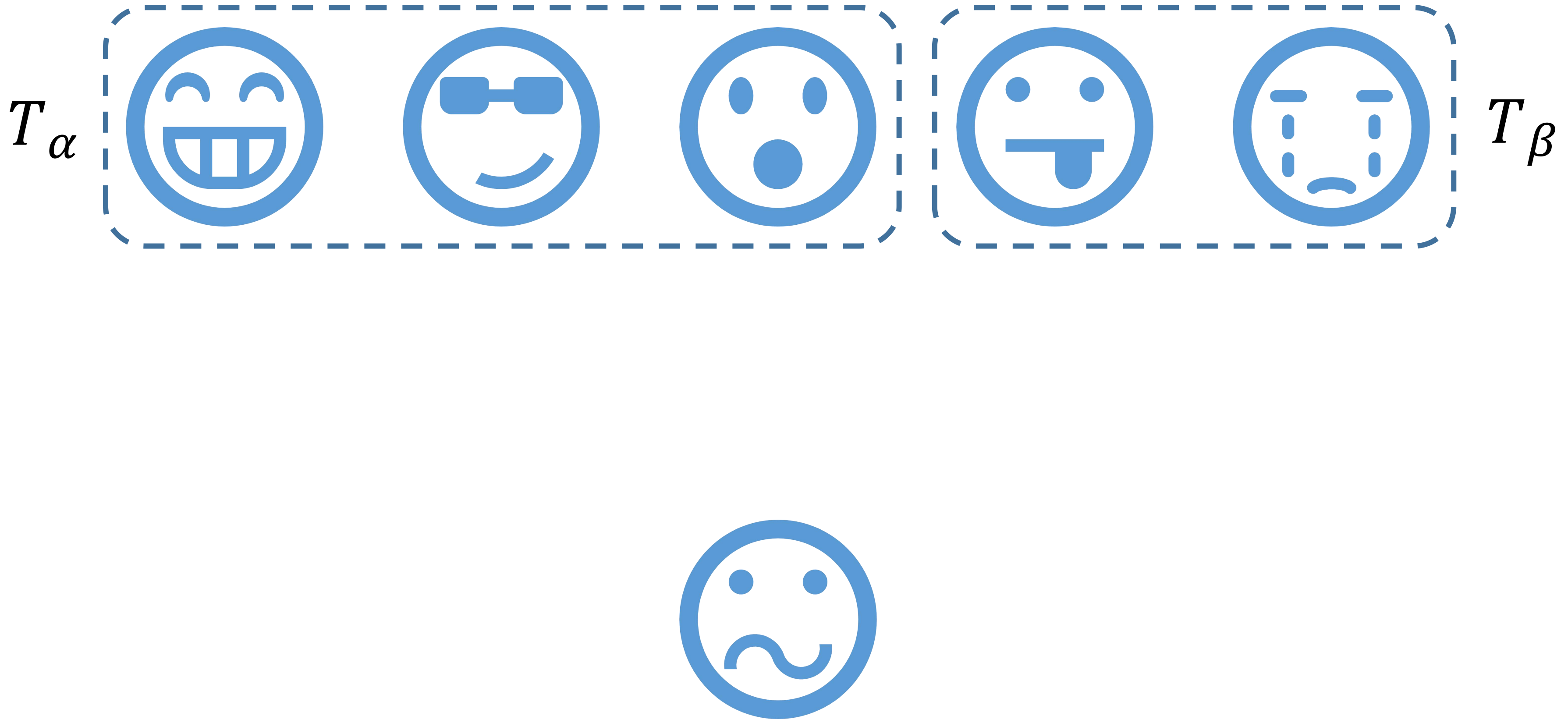


Aggregated Observation

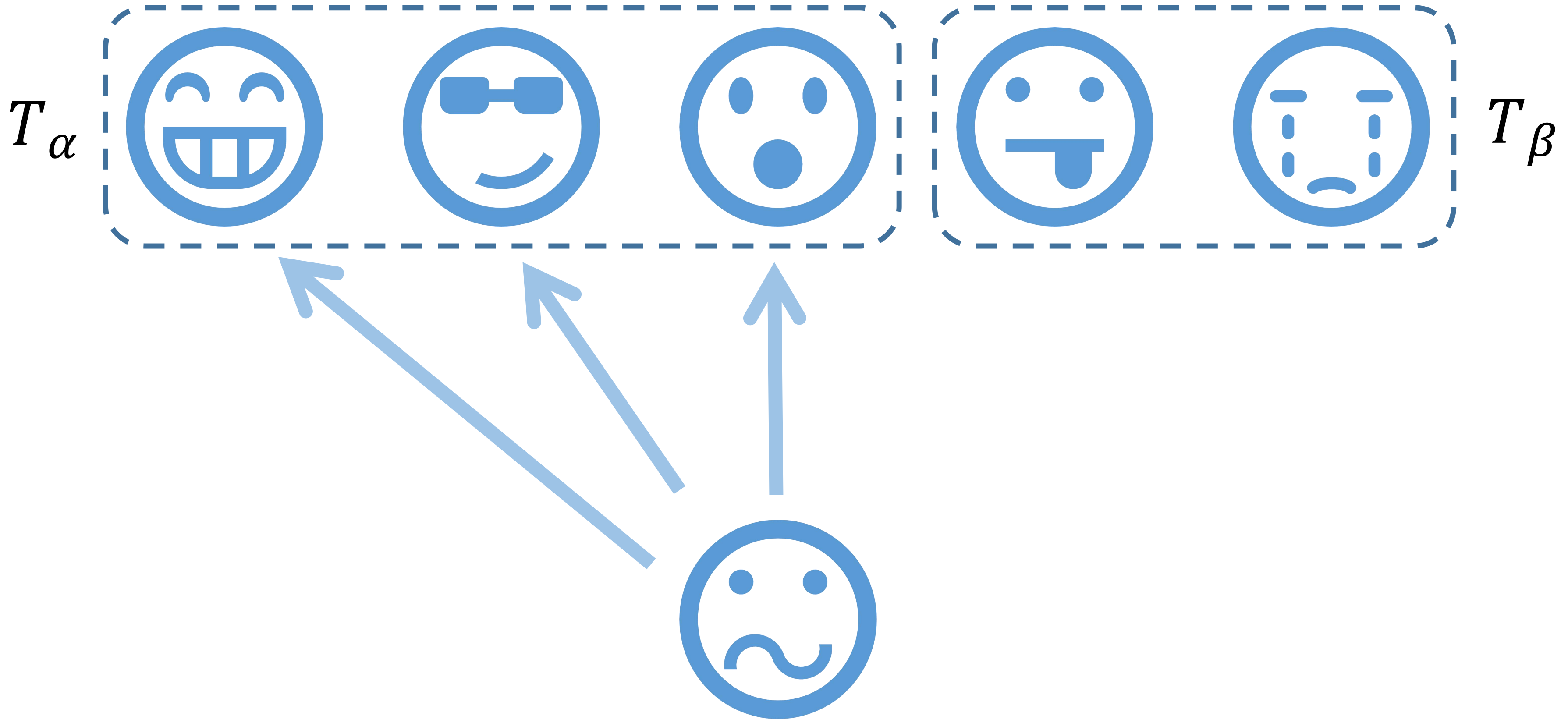
Binary Search



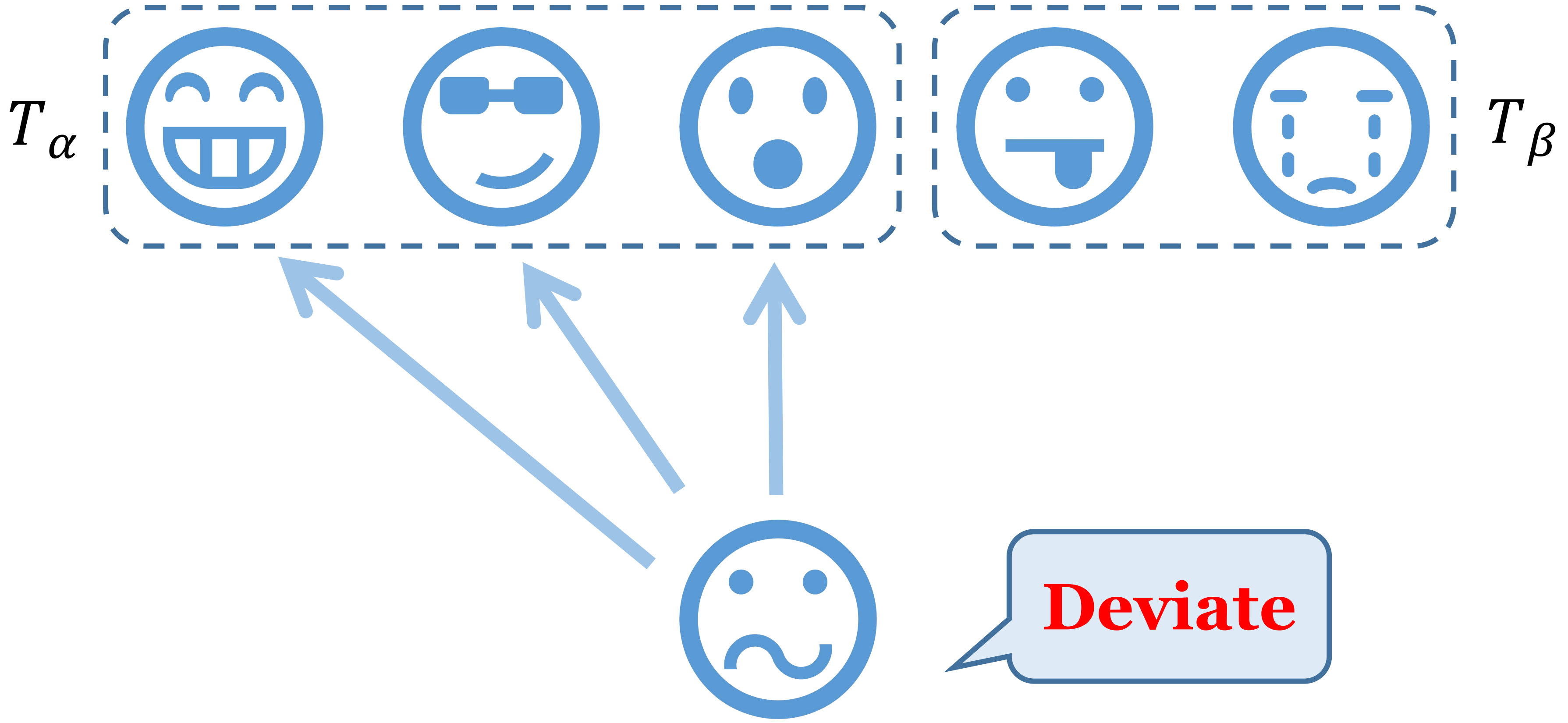
Binary Search



Binary Search



Binary Search



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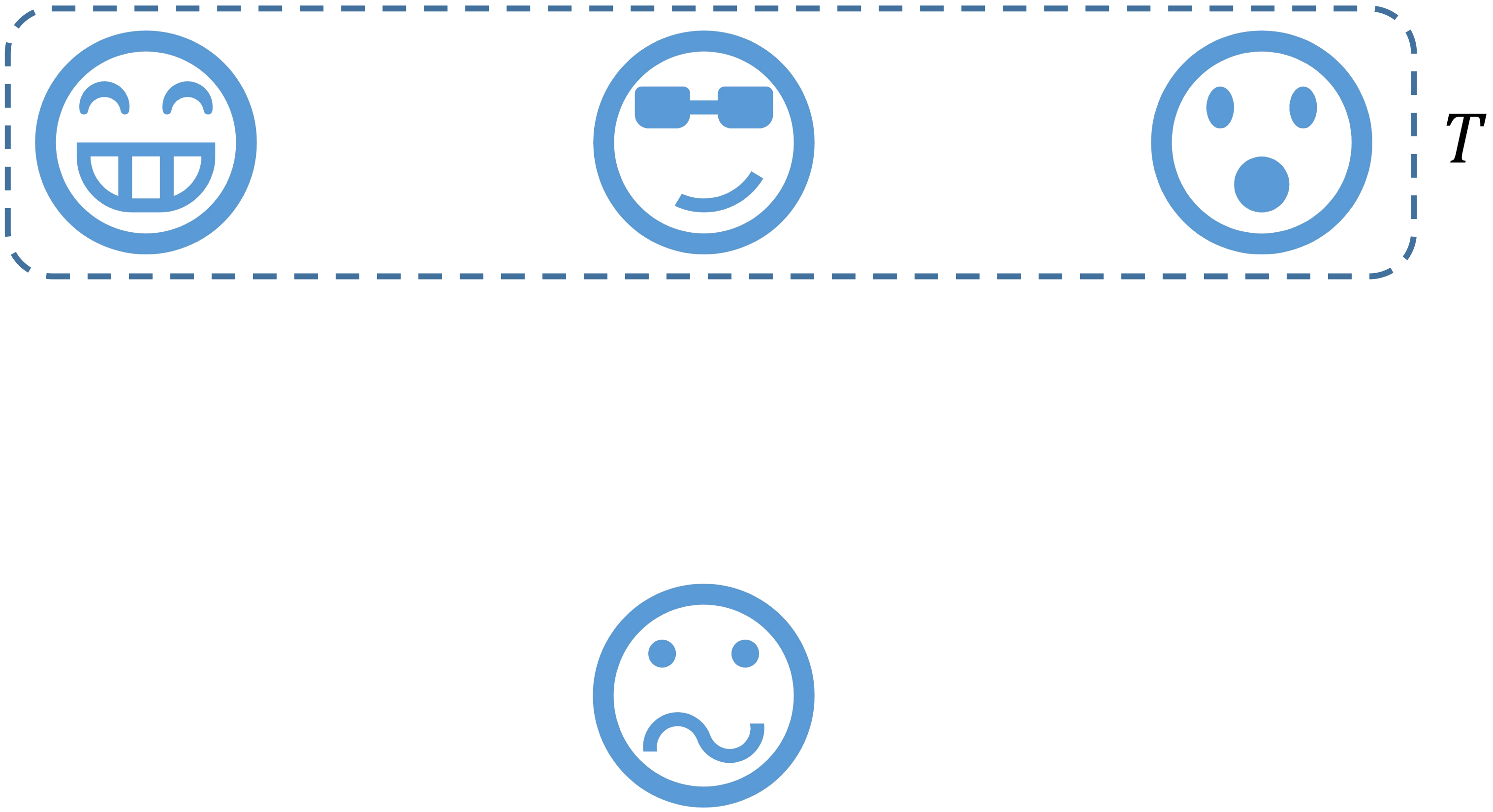

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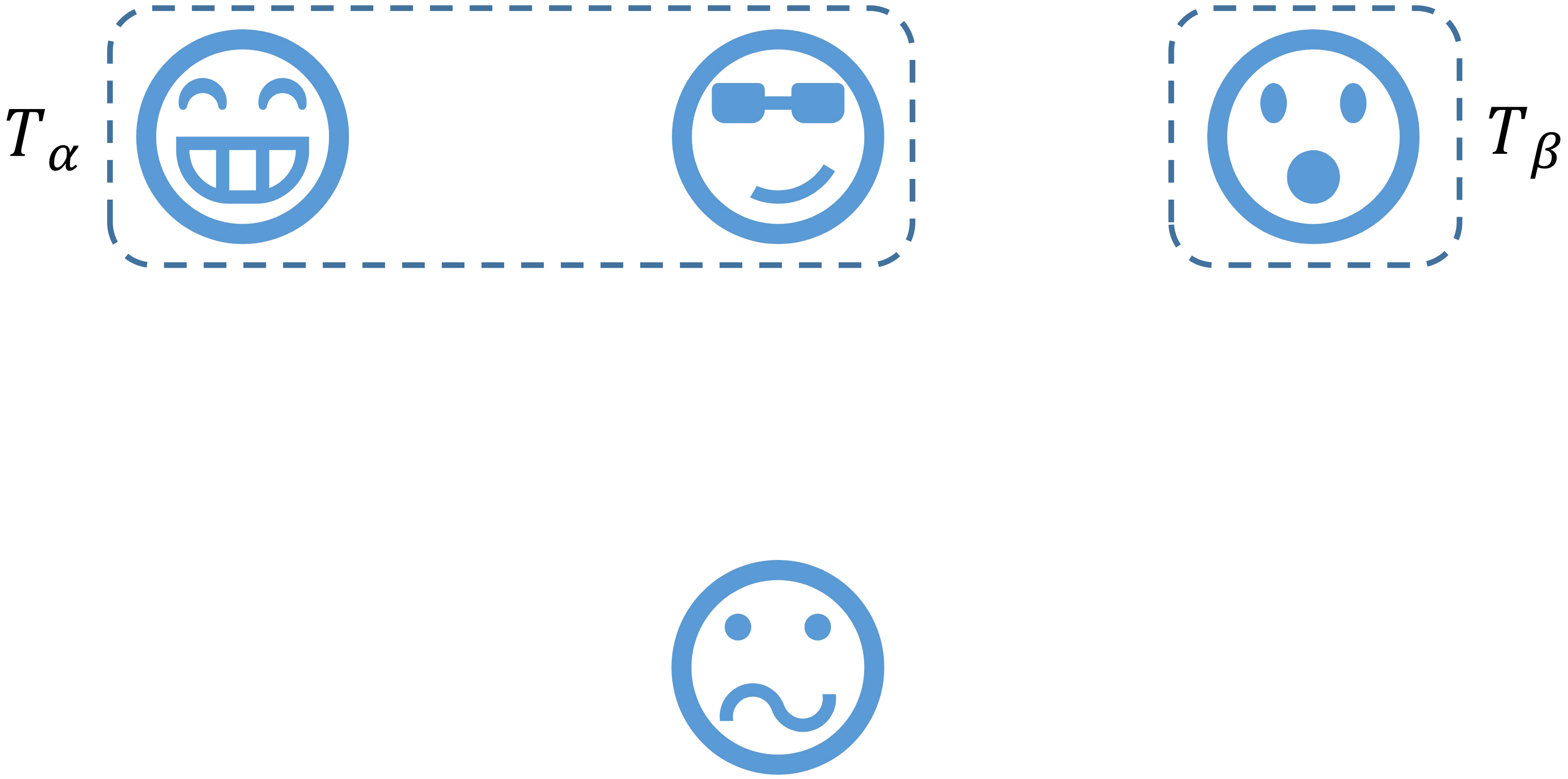

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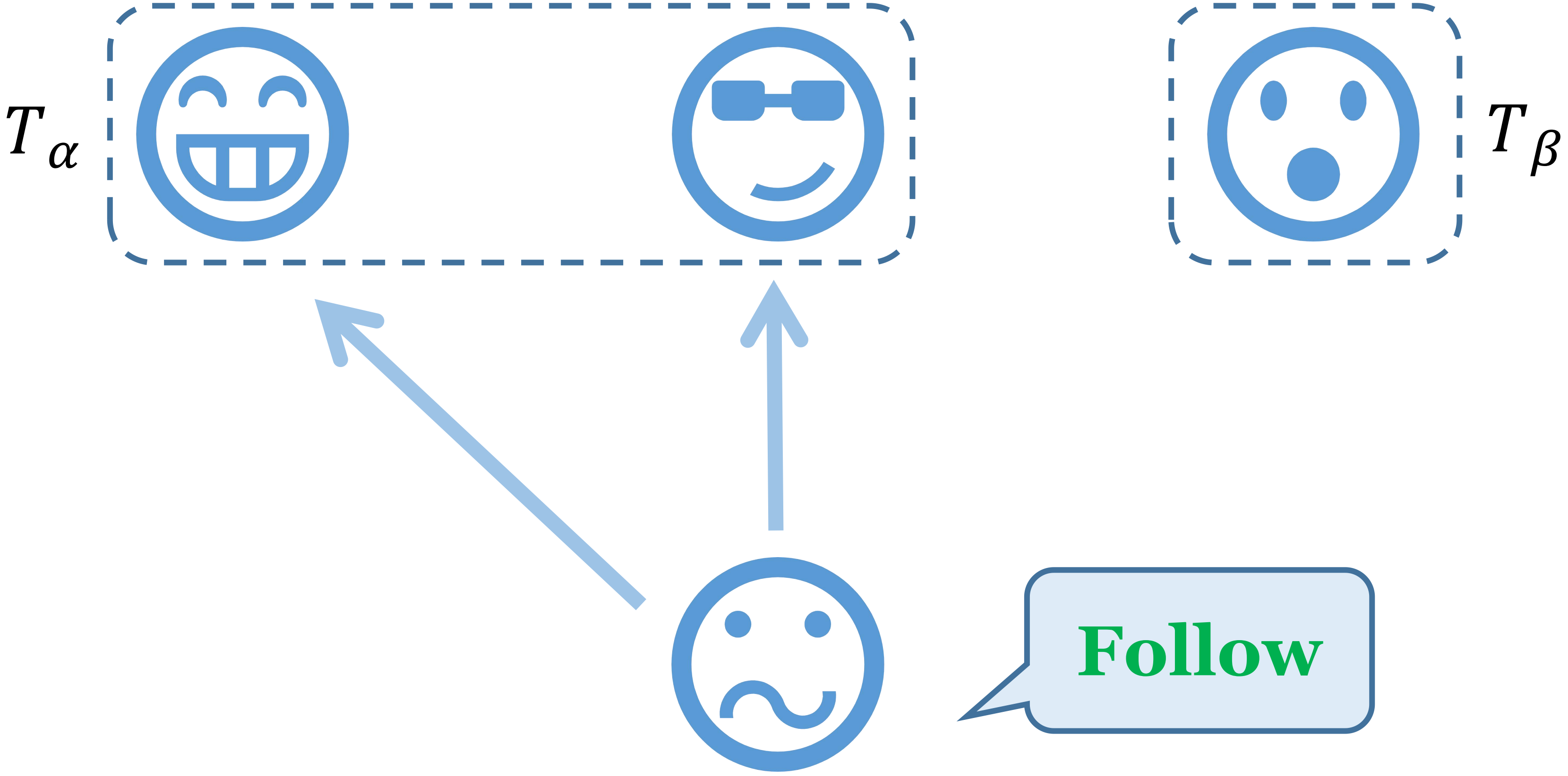

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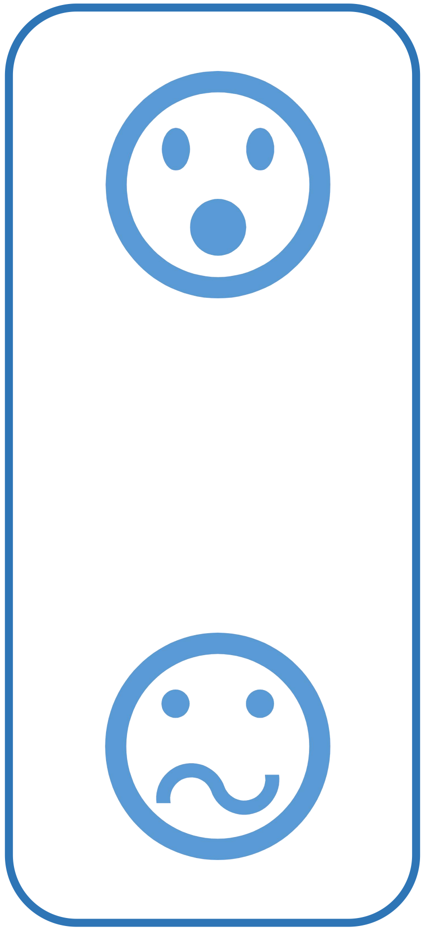
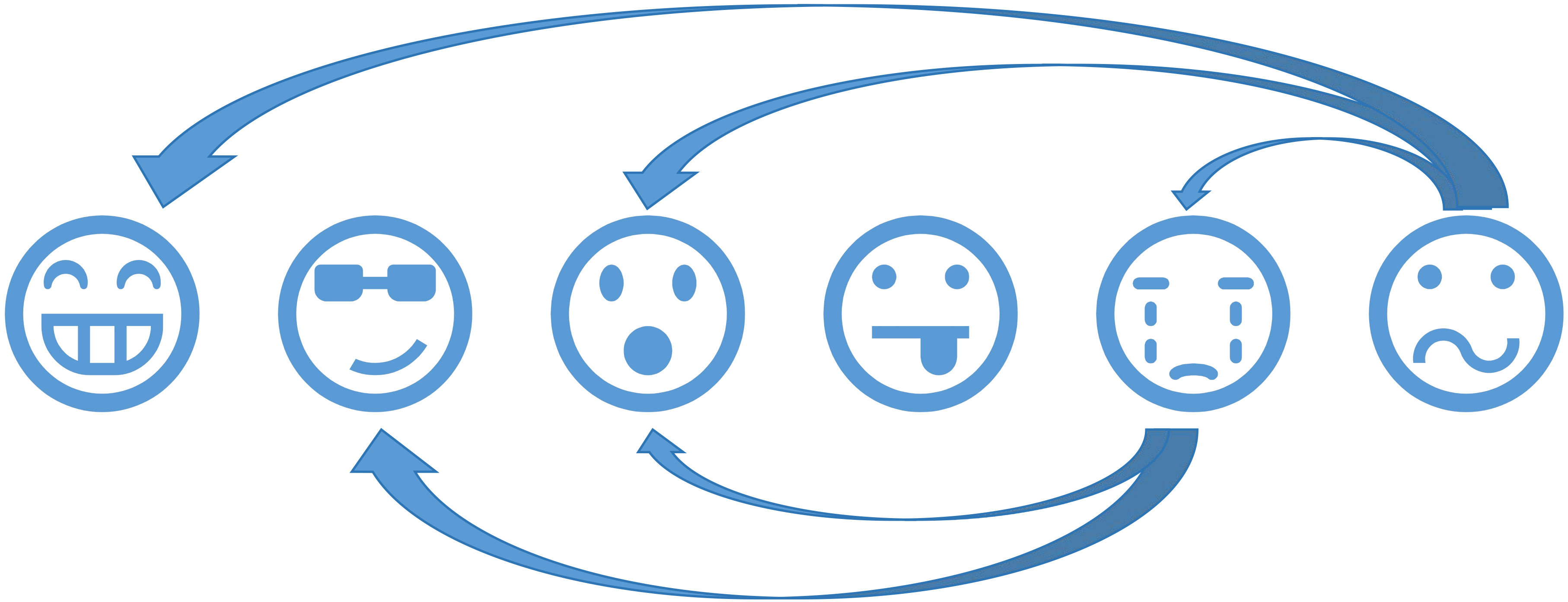
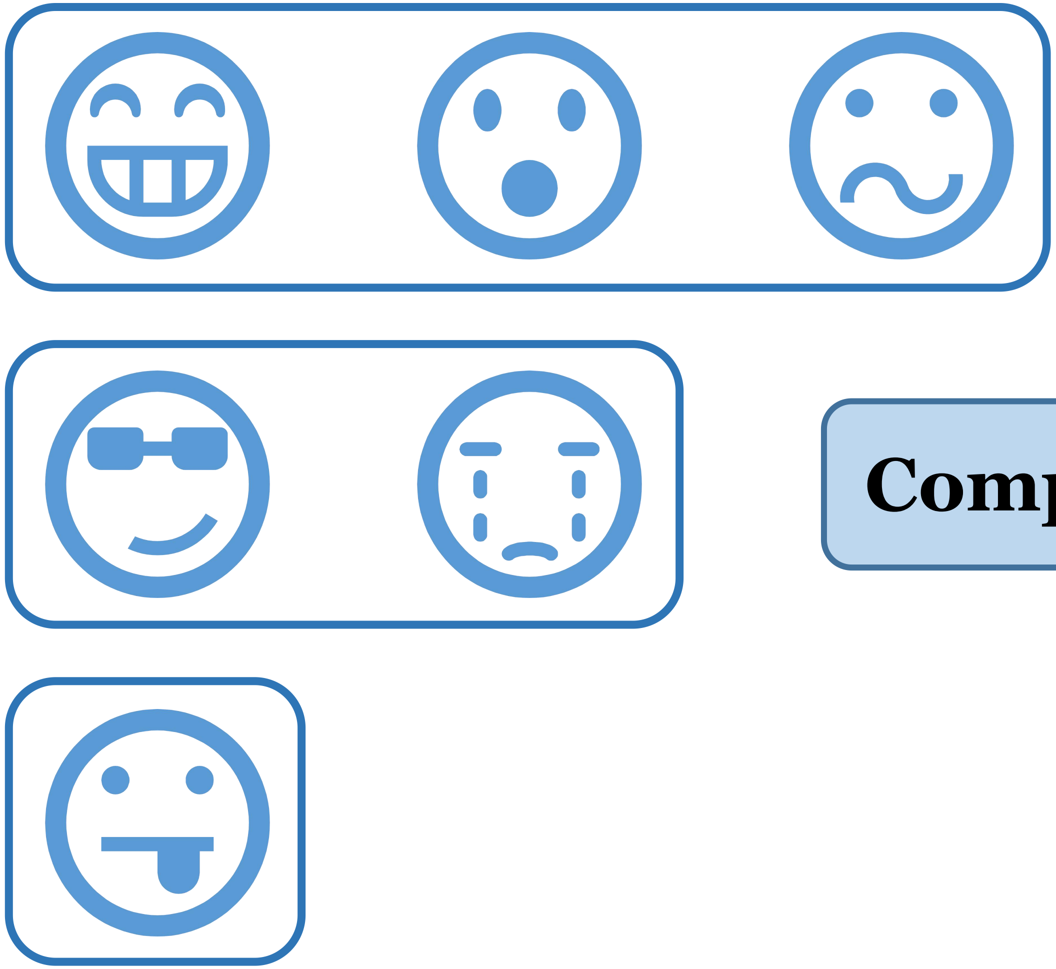


Simultaneous Binary Search



Simultaneous Binary Search



Complexity: $\log_2 n + 2$

Other Types of Games

	Congestion	Graphical	Auctions
Lower Bound*	$\log_2 n$	$\max(\log_2 n, n/d)$	$\log_2 n$
Algorithm	$\log_2 n + 2$	$2n/d + 2\log_2 d + 1$	$(1 + \log_2 n)(1 + c) + 1$
Technique	Directed Brass's paradox	Block decomposition	Bitwise search

- **In the above:**
  - d is the degree of the graph
  - c is the size of the largest coalition
- Mostly, the algorithm's **complexity matches the lower bound**

Our Contributions

- We study the CSL problem with **multiple-bit observations**
  - Since single-bit ones lead to **super-linear** rounds of interaction
- We conduct a thorough theoretical study
  - We present **algorithms** that learn the coalition structure with a **sub-linear** number of rounds using different types of games
  - We complement our algorithmic results with **lower bounds** that shows their **optimality** in most settings we consider

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