# Deviate or Not: Learning Coalition Structures with Multiple-bit Observations in Games


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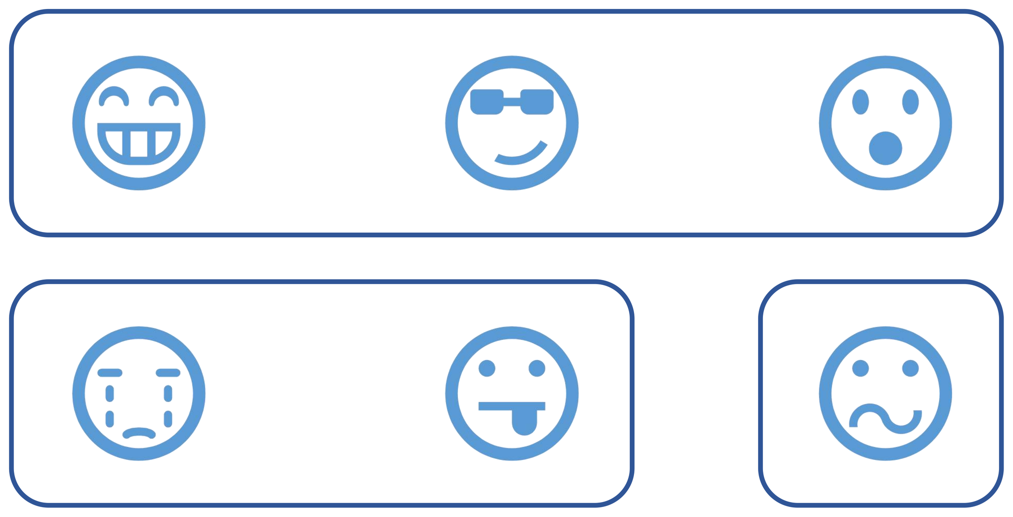
## Coalition Structure Learning (CSL)

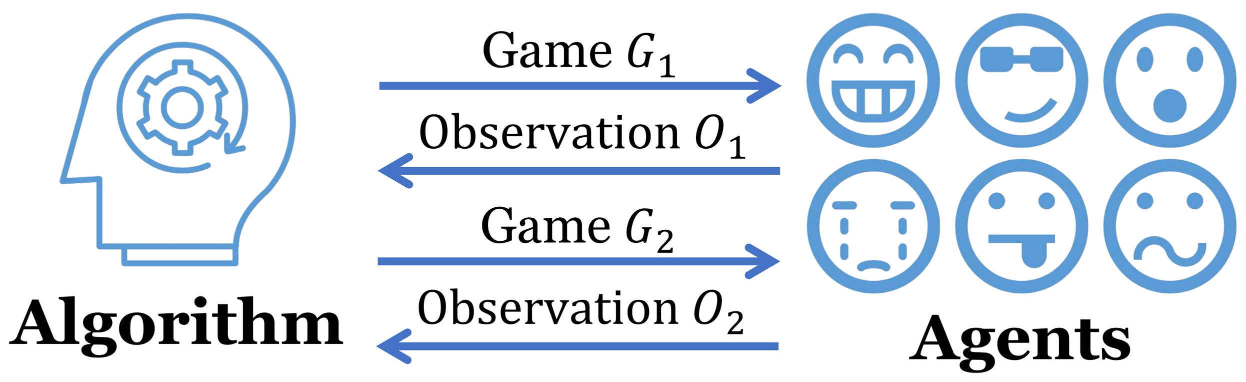


**Coalition:** A nonempty subset of the agents, in which the agents **coordinate their actions** and **have common interests**.

**Coalition Structure:** A set partition of the agents $\{1, 2, \cdots, n\}$

**Behavior Model in a Game:** Each coalition **act as a joint player** whose actual utility equals the **total utilities of its members**

**Coalition Structure Learning (CSL):** Recover the unknown coalition structure by observing interactions in designed games




*What kind of **games** can the algorithm design?*

*What **observation** does the algorithm obtain?*

**Single-Bit Observation Oracle:** The algorithm queries a game $G$ and a strategy profile $\Sigma$, the agents answer whether $\Sigma$ is an **NE** in $G$

Easy to compute for the agents, **one bit of information** per query

**Theorem (Xu et al. 2024):** **Any algorithm** for CSL must interact **at least** $n \log_2 n - O(n \log_2 \log_2 n)$ rounds with the agents

**Multiple-Bit Observation Oracle:** The algorithm queries a game $G$ and a strategy profile $\Sigma$, each agent indicates **whether to deviate**

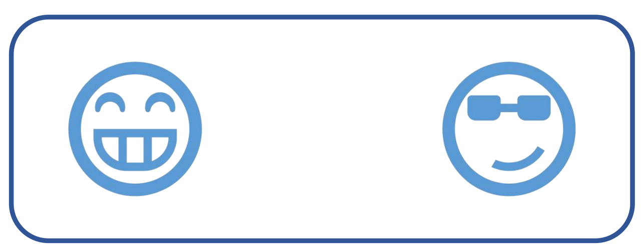
Still Easy to compute, **$n$ bits of information** per query

Reduces the lower bound to **$\log_2 n - O(\log_2 \log_2 n)$**

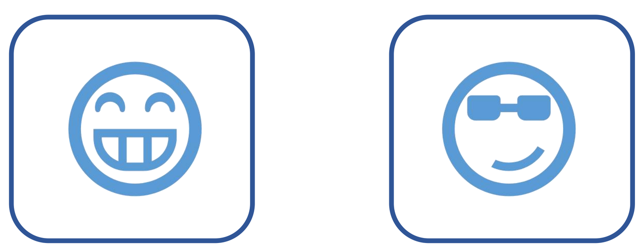
Opens up the possibility of much more efficient algorithms

**Types of Games:** **Normal form games**, **congestion games**, **graphical games**, **auctions**. We study **all** four settings in this paper, and show **asymptotically optimal algorithms** for most of them.

## Normal-form: Directed Prisoner's Dilemma



*How to distinguish between the two?*



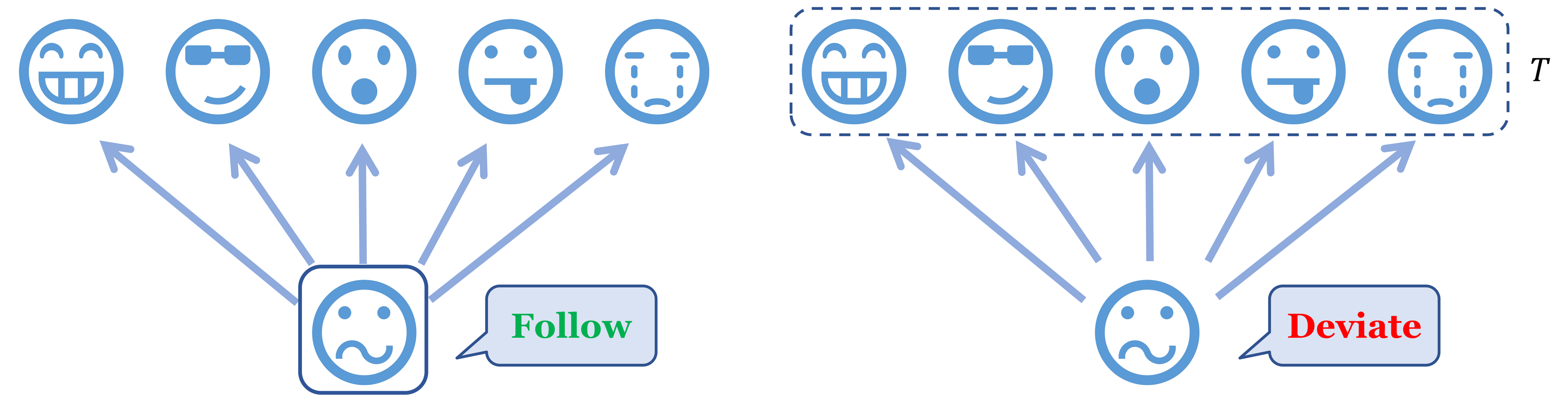
**Directed Prisoner's Dilemma:** A normal form game between agents $(x, y)$, where agent $y$ can choose cooperate, losing 1 unit of utility and giving agent $x$ 2 units of utility

| | $C_y$ | $D_y$ |
|---|---|---|
| $D_x$ | (2, -1) | (0, 0) |

**Lemma:** If we specify $(D_x, D_y)$ as $\Sigma$, then agent $y$ **deviates** from $D_y$ if and only if $x$ and $y$ are **in the same coalition**

## Aggregated Observation

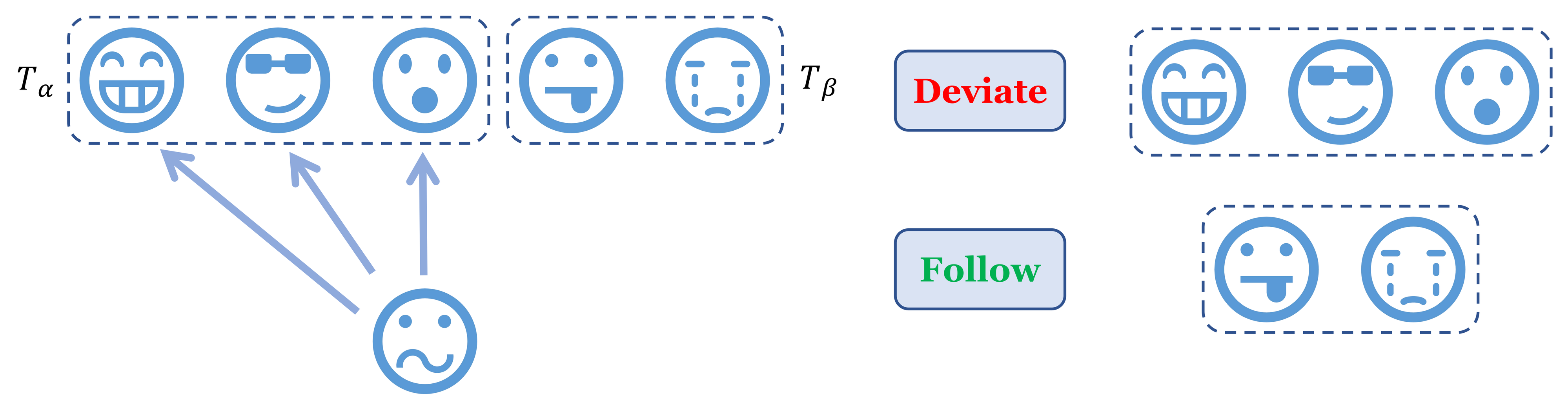
If we let agent $y$ play Directed Prisoner's Dilemmas against all others



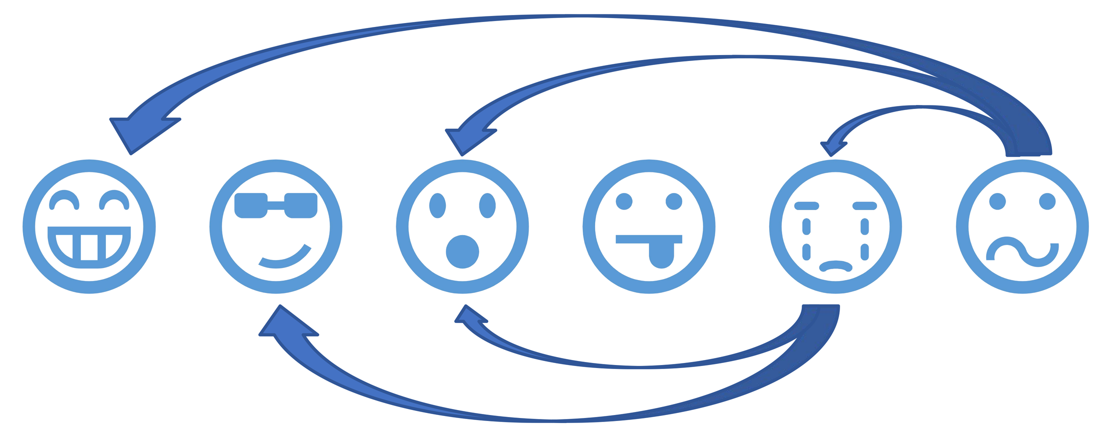

*Agent $y$ is NOT cooperating with others* *Agent $y$ IS cooperating with others*

## Simultaneous Binary Search

If agent $y$ is cooperating with others, we can use **binary search** to find the agent with **the smallest index** within agent $y$'s coalition




This binary search can be done **simultaneously** for each agent



**Theorem:** Simultaneous Binary Search solves CSL in **$\log_2 n + 2$** rounds.

## Solving CSL with Other Types of Games

We summarize the results for CSL using other types of games below

| | Congestion | Graphical | Auctions |
|---|---|---|---|
| Lower Bound* | $\log_2 n$ | $\max(\log_2 n, n/d)$ | $\log_2 n$ |
| Algorithm | $\log_2 n + 2$ | $2n/d + 2\log_2 d + 1$ | $(1 + \log_2 n)(1 + c) + 1$ |
| Technique | Directed Brass's paradox | Block decomposition | Bitwise search |

**In the above:**

$d$ is the degree of the graph

$c$ is the size of the largest coalition

Mostly, the algorithm's **complexity matches the lower bound**