Non-excludable Bilateral Trade between Groups

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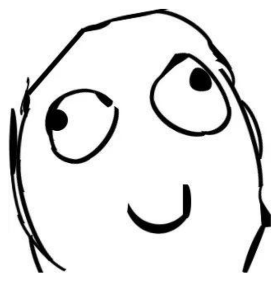
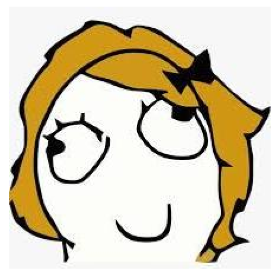
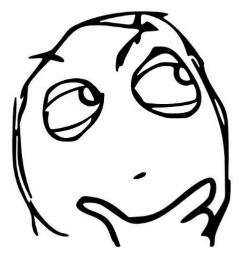
Bilateral Trade (Classic Setting)



Buyer
Private **value**:
$v \sim F$



Seller
Private **cost**:
$c \sim G$



Public:
F, G

Based on **interactions** with the players, a **mechanism** decides:
Whether to **trade** x, **payment** of the buyer p, **receipt** of the seller r.

Revelation principle: WLOG, **interactions** can be viewed as a sealed **bid** b from the buyer and a sealed **ask** a from the seller.

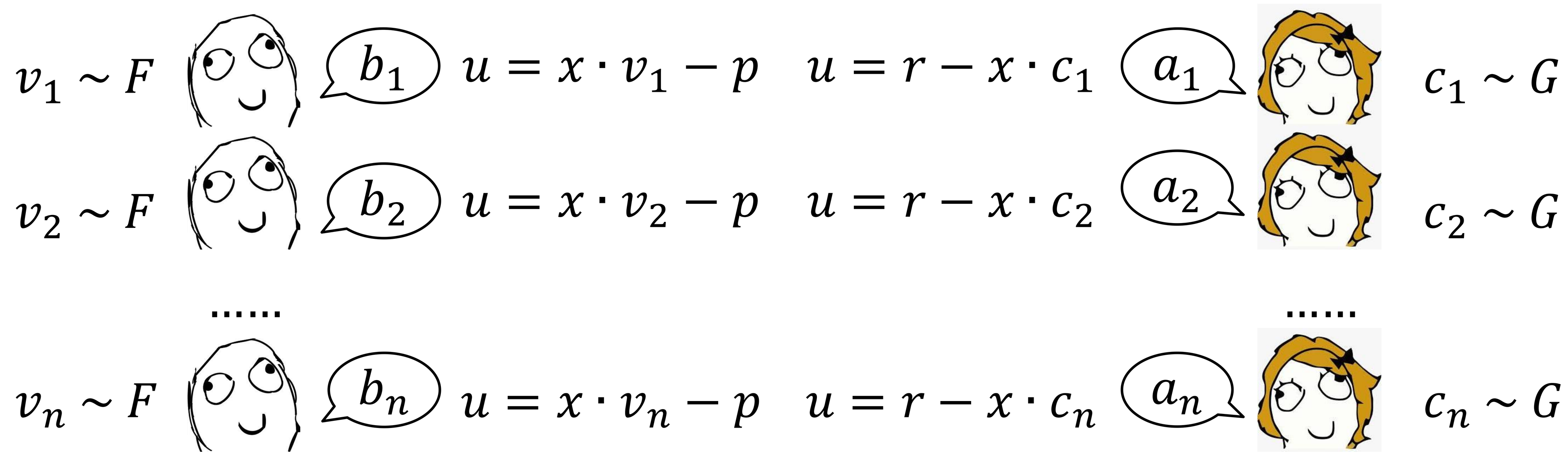
A mechanism: $\{x(a,b), p(a,b), r(a,b)\}$
Utilities: $u_b(a,b) = v \cdot x(a,b) - p(a,b), u_s(a,b) = r(a,b) - c \cdot x(a,b)$

Desiderata of a mechanism:

- **Incentive compatible (IC):** Players bid and ask truthfully
- **Individually rational (IR):** Players' utilities are non-negative
- **Budget balanced (BB):** Buyer's payment ≥ seller's receipt
- **Efficient:** A trade happens whenever $v > c$

Myerson and Satterthwaite (1983): It is impossible to achieve all of {IC, IR, BB, Efficient} in bilateral trade, i.e, ***efficient bilateral trade cannot be implemented in a feasible way***.

Group Bilateral Trade (Our Setting)



Group Trading: We consider a richer paradigm, with many buyers and sellers on both sides of a trade, hoping to bypass the impossibility.

Non-Excludability: the mechanism guarantees

- The **players share** the same **allocation** x
- The **buyers share** the same **payment** p
- The **sellers share** the same **receipt** r

Desiderata of a mechanism:

- **Incentive compatible (IC):** Players bid and ask truthfully
- **Individually rational (IR):** Groups' utilities are non-negative
- **Budget balanced (BB):** Buyer's payment ≥ seller's receipt
- **Efficient (in the limit):** As $n \to \infty$, GFT/FB $\to 1$

Asymptotics: Real life intuition shows that although negotiation between individuals are inefficient, that of two sizeable organizations is usually better. Thus, we treat n as the only asymptotic variable, and let $n \to \infty$. Note that when $n = 1$, we recover the classic setting.

Our Results

A **dichotomy** in the possibility of trading efficiently.
If the buyers value the item (strictly) more than the sellers:

- A mechanism achieving all desiderata in the limit is given.

If the sellers value the item (weakly) more than the buyers:

- No mechanisms can achieve all desiderata in the limit.

Both **deterministic** ($x(\boldsymbol{b}, \boldsymbol{a}) \in \{0,1\}$), and **smooth randomized** ($x(\boldsymbol{b}, \boldsymbol{a}) \in [0,1]$, twice continuously differentiable) mechanisms are studied.

Why Two Cases?

Lemma 4.1. Consider the **first best (FB)** in both cases.

a) If $E_{v\sim F}[v] > E_{c\sim G}[c]$, then FB $= \Omega(n)$.
b) If $E_{v\sim F}[v] \le E_{c\sim G}[c]$, then FB $= O(\sqrt{n})$.

Lemma 4.1 naturally divides the problem into two cases. When the sellers value item more, even FB goes to zero (per agent).

Characterization of IC Mechanisms

Theorem 4.1. Deterministic allocation $x(\boldsymbol{b}, \boldsymbol{a})$ can be implemented by an **IC** deterministic mechanism **if and only if:**

a) For any $\boldsymbol{a}$, there is $\tau_{\boldsymbol{a}}$ and a monotone Boolean function $f_{\boldsymbol{a}}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = f_{\boldsymbol{a}}(\mathbf{1}[b_1 \ge \tau_{\boldsymbol{a}}], \mathbf{1}[b_2 \ge \tau_{\boldsymbol{a}}], \ldots, \mathbf{1}[b_n \ge \tau_{\boldsymbol{a}}])$
b) For any $\boldsymbol{b}$, there is $\theta_{\boldsymbol{b}}$ and a monotone Boolean function $g_{\boldsymbol{b}}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = g_{\boldsymbol{b}}(\mathbf{1}[a_1 \le \theta_{\boldsymbol{b}}], \mathbf{1}[a_2 \le \theta_{\boldsymbol{b}}], \ldots, \mathbf{1}[a_n \le \theta_{\boldsymbol{b}}])$

Informally: An IC mechanism should decide in a **voting-like** way.

Theorem 5.1. Smooth randomized allocation $x(\boldsymbol{b}, \boldsymbol{a})$ can be implemented by an **IC** randomized mechanism **if and only if:**

a) For any $\boldsymbol{a}$, there are n non-decreasing differentiable functions $f_{\boldsymbol{a},i}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = f_{\boldsymbol{a},1}(b_1) + f_{\boldsymbol{a},2}(b_2) + \ldots + f_{\boldsymbol{a},n}(b_n)$
b) For any $\boldsymbol{b}$, there are n non-increasing differentiable functions $g_{\boldsymbol{b},i}$, such that $x(\boldsymbol{b}, \boldsymbol{a}) = g_{\boldsymbol{b},1}(a_1) + g_{\boldsymbol{b},2}(a_2) + \ldots + g_{\boldsymbol{b},n}(a_n)$

Informally: An IC mechanism must be **separable** across agents.

Buyers Value More: Positive Result

Algorithm 1: Always trade at price $\frac{1}{2}(E_{v\sim F}[v] + E_{c\sim G}[c])$.

Theorem 4.3. Algorithm 1 is **IC** and **SBB**. When $E_{v\sim F}[v] > E_{c\sim G}[c]$, w.p. $1 - e^{-\Omega(n)}$, it is **IR**, and its **efficiency** is $1 - e^{-\Omega(n)}$.

Informally: Algorithm 1 achieves all desiderata in the limit (in this case).

Sellers Value More: Negative Result

Theorem 4.4. When $E_{v\sim F}[v] \le E_{c\sim G}[c]$, **no deterministic IC** mechanisms can be **efficient** in the limit.

Theorem 5.2. When $E_{v\sim F}[v] \le E_{c\sim G}[c]$, **no smooth randomized IC** mechanisms can be a **constant approximation of FB** in the limit.